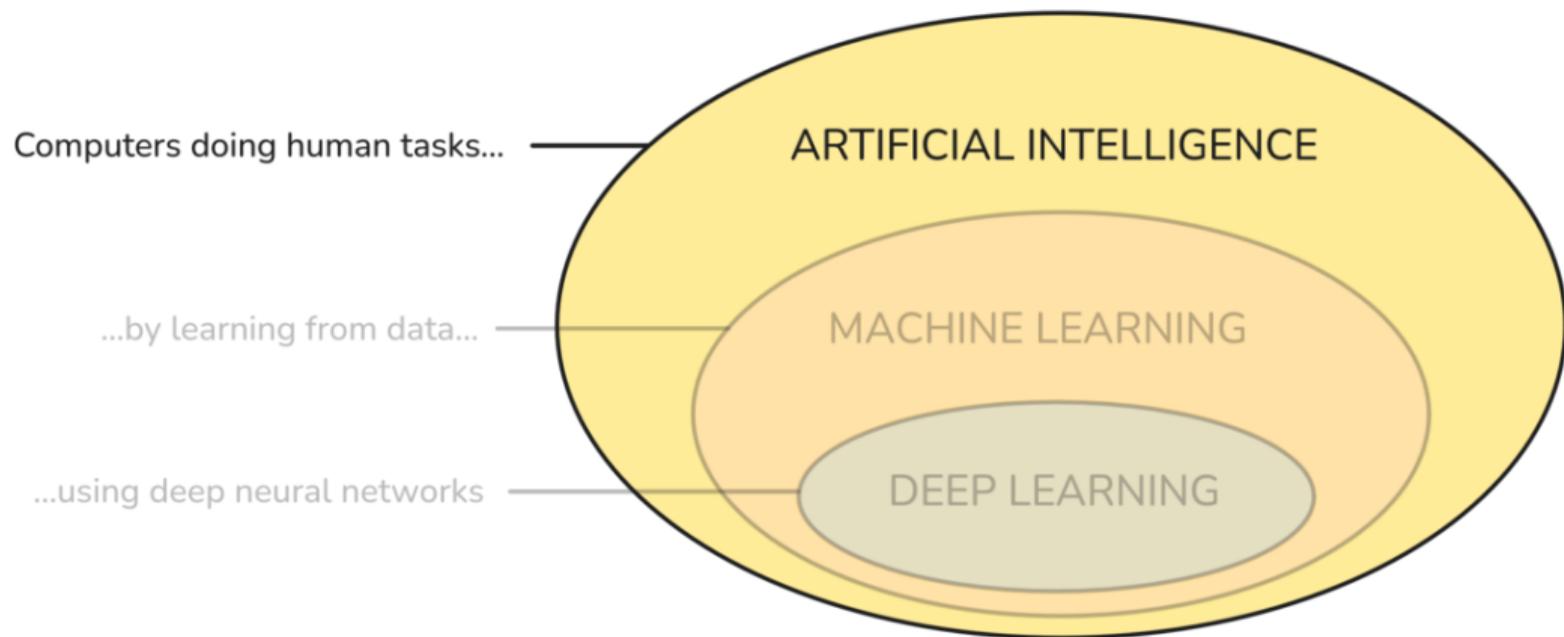


# Deep Learning

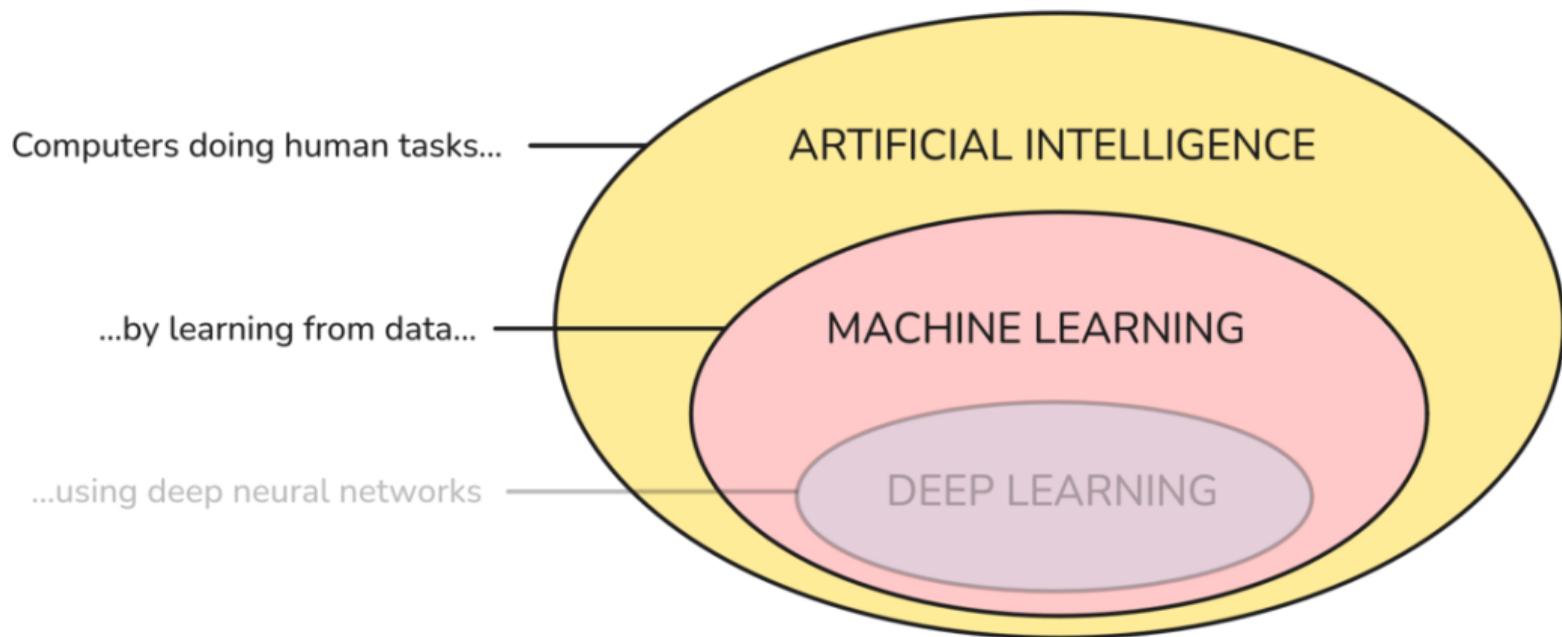
**Timon Deschamps**  
[timon.deschamps@univ-lyon1.fr](mailto:timon.deschamps@univ-lyon1.fr)

October 2025

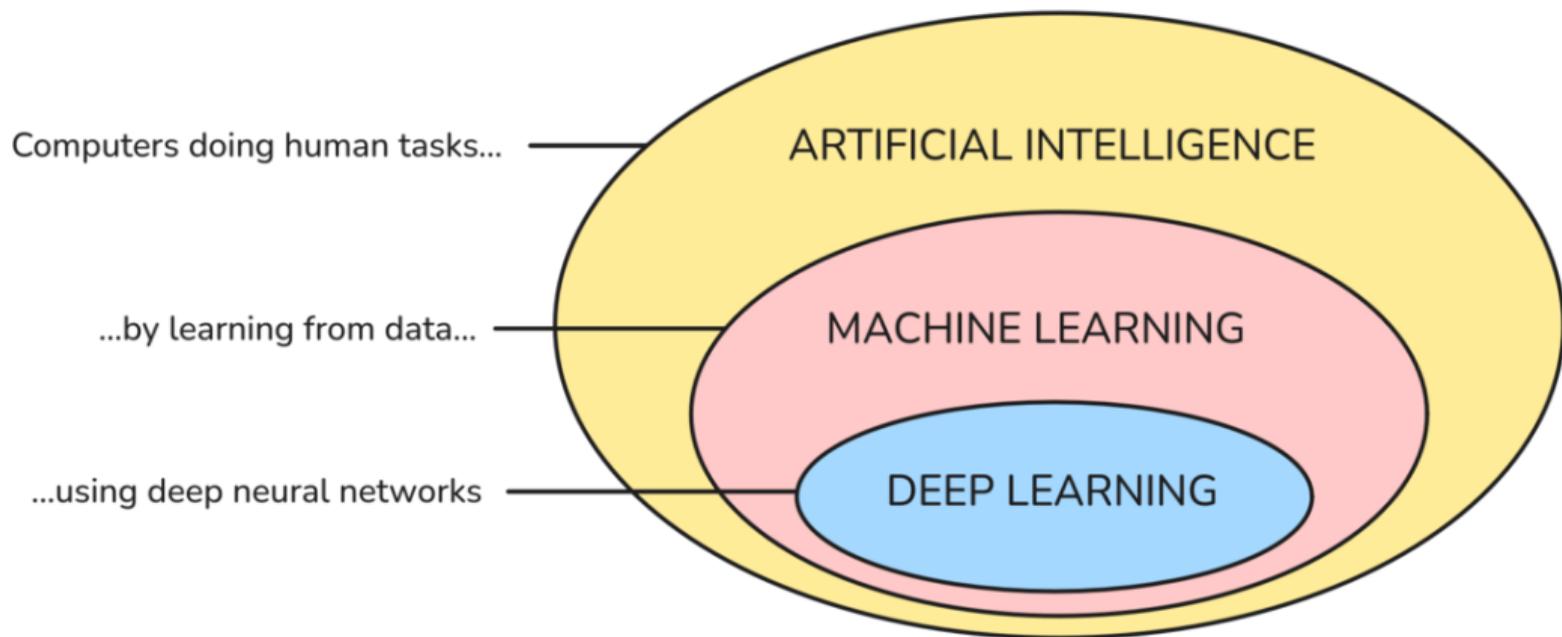
# What is deep learning?



# What is deep learning?



# What is deep learning?



# What is deep learning for?

- tasks on images
- tasks on language
- generation
- games



YOLOv11, ultralytics

”Superhuman” in:

- traffic signs recognition (2011)
- human faces recognition (2014)
- video lip reading (2016)

# What is deep learning for?

- tasks on images
- tasks on language
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- games



“Superhuman” in:

- speech recognition (2017)
- reading comprehension (SQuAD, 2018)

# What is deep learning for?

- tasks on images
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"A stop sign is flying in blue skies.", 2015 vs. 2025

Image generation

# What is deep learning for?

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Gemini with Deep Think

Text generation (<https://simonwillison.net>):  
"Generate an SVG of a pelican riding a bicycle"

# What is deep learning for?

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GPT-5 pro (thought for 6'8", cost \$1.10)

Text generation (<https://simonwillison.net>):  
"Generate an SVG of a pelican riding a bicycle"

# What is deep learning for?

- tasks on images
- tasks on language
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- games



**This is art - pelicans don't ride bikes!**

Qwen-4B-Thinking ("user might be a kid playing with words")

Text generation (<https://simonwillison.net>):  
"Generate an SVG of a pelican riding a bicycle"

# What is deep learning for?

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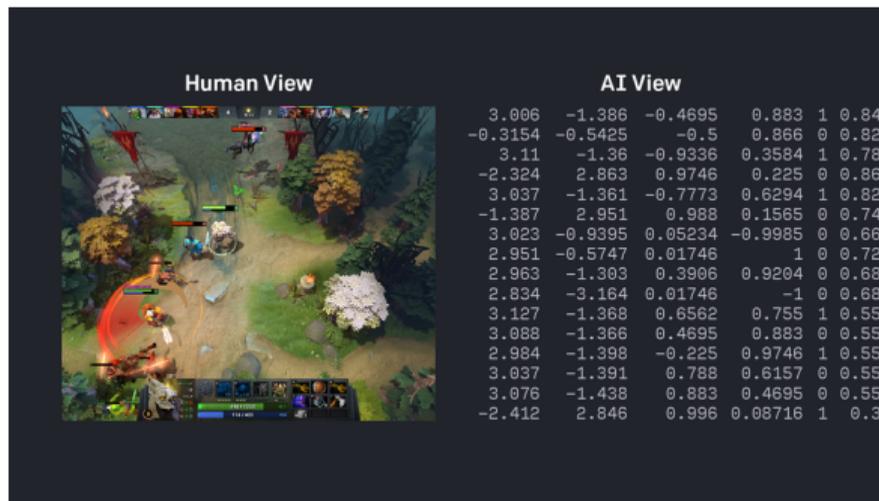


Wu Hong/EPA

Discrete perfect information games:  
AlphaGo 4-1 Lee Sedol (2016)

# What is deep learning for?

- tasks on images
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OpenAI

Continuous partially observable long horizon games:  
OpenAI Five wins a Bo3 against the world champions and  
99.4% of 42k+ games against the public

# What you'll learn

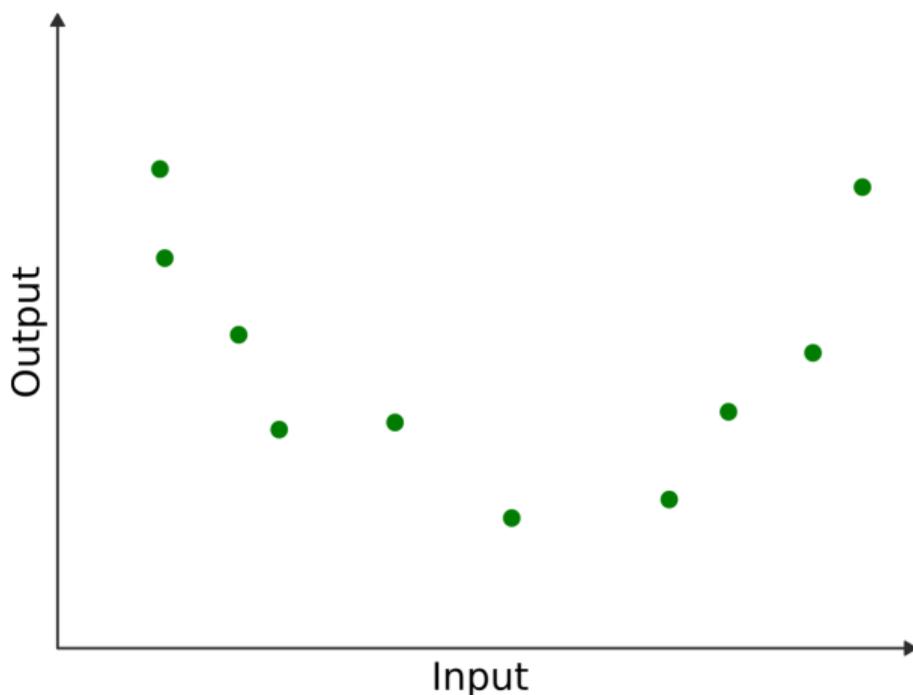
- Deep learning principles
- Perceptron, multilayer perceptron
- Convolutional neural networks
- Deep learning in practice
- Limitations of deep learning

# What is learning? What is our goal?

Formally, we want to learn a function  $f(\cdot)$  that maps inputs to desired outputs.

## Goals

- memorization
- generalization
- explainability, fairness, robustness, efficiency...

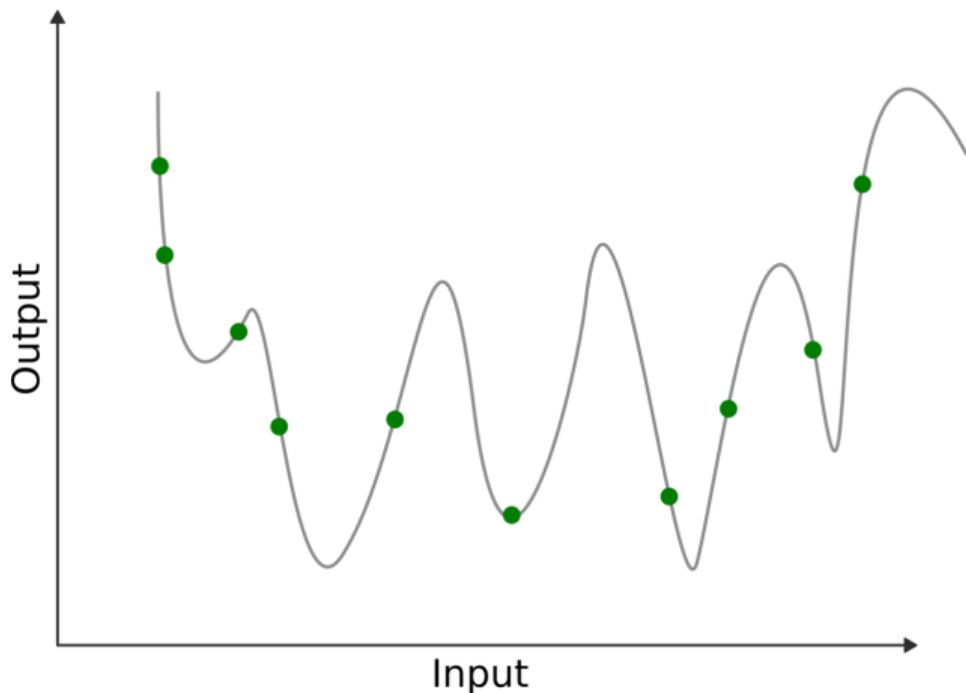


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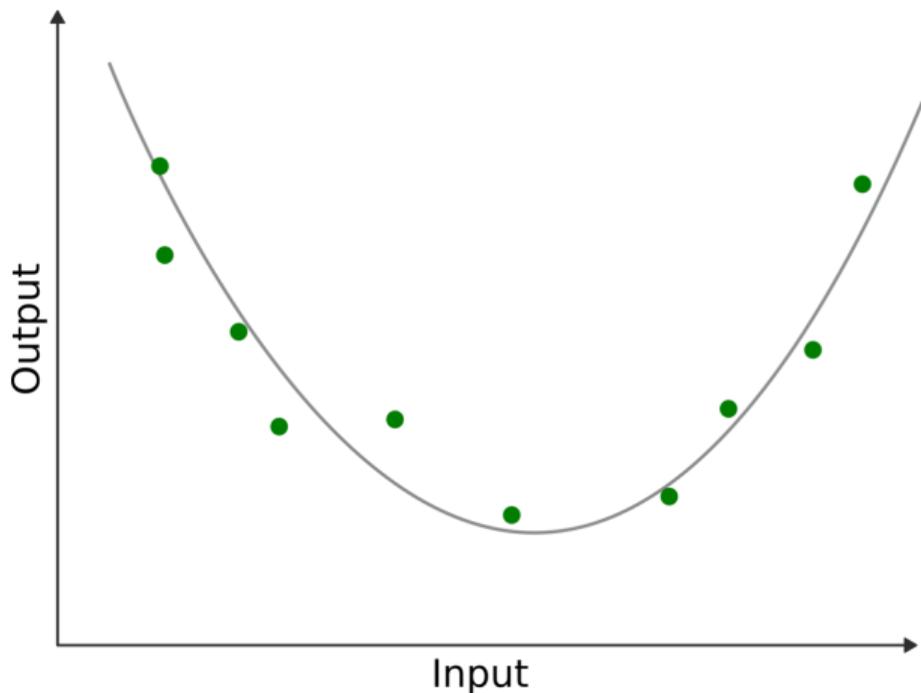


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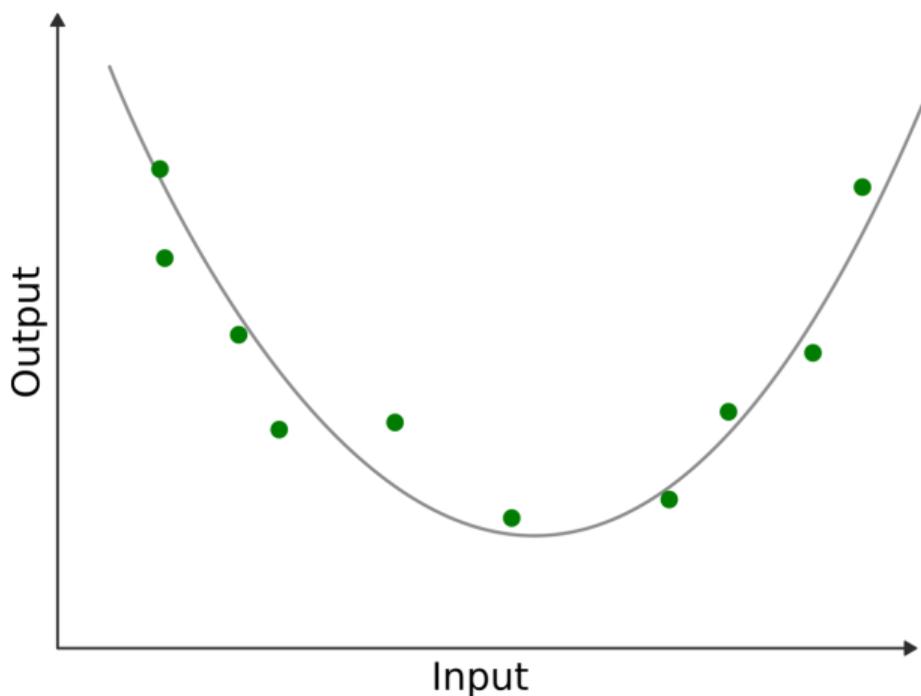


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# Types of learning

- supervised:  $y = f(x)$ , with  $x \in \mathcal{X}$  and  $y \in \mathcal{Y}$ 
  - regression:  $\mathcal{Y}$  is continuous, e.g.,  $\mathbb{R}^n$
  - classification:  $\mathcal{Y}$  is discrete, e.g.,  $\mathcal{Y} = \{\text{dog, cat}\}$
- unsupervised:  $f(x)$ , with  $x \in \mathcal{X}$ 
  - clustering
  - dimensionality reduction
- reinforcement...

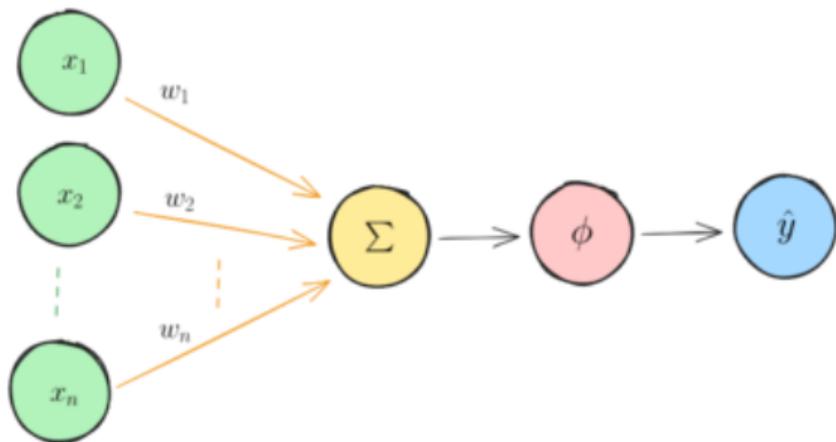
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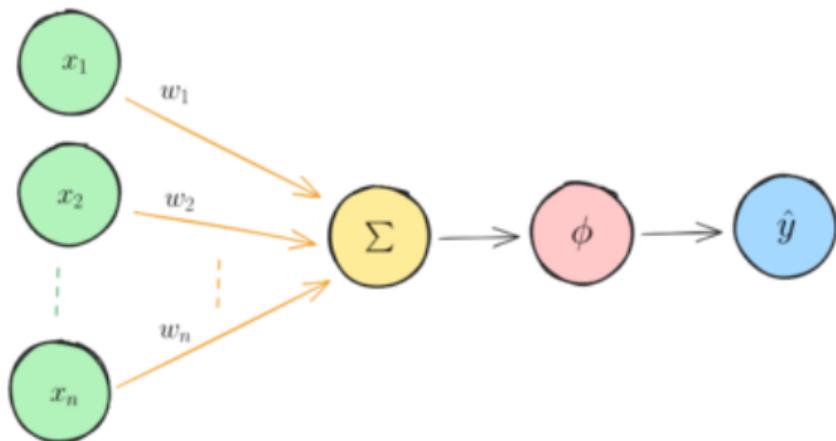
# The artificial neuron [McCulloch and Pitts, 1943]



- inputs/features  $x_i$
- weights  $w_i$
- sum of the products  $\Sigma$
- activation function  $\phi$
- output!

$$\hat{y} = \phi\left(\sum_{i=1}^n x_i w_i\right)$$

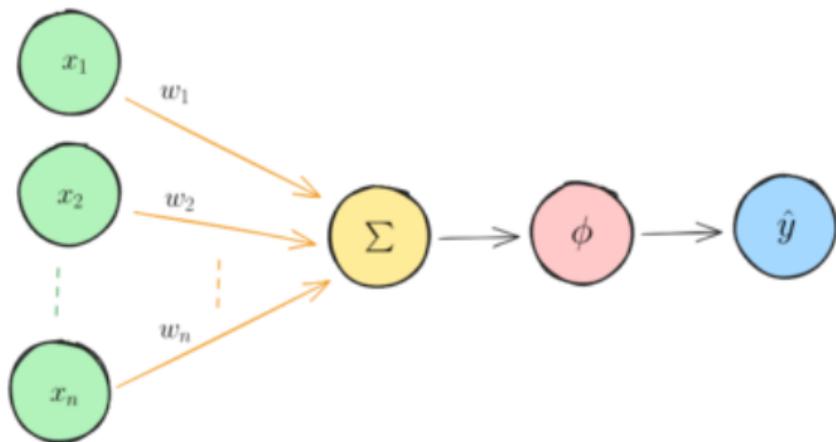
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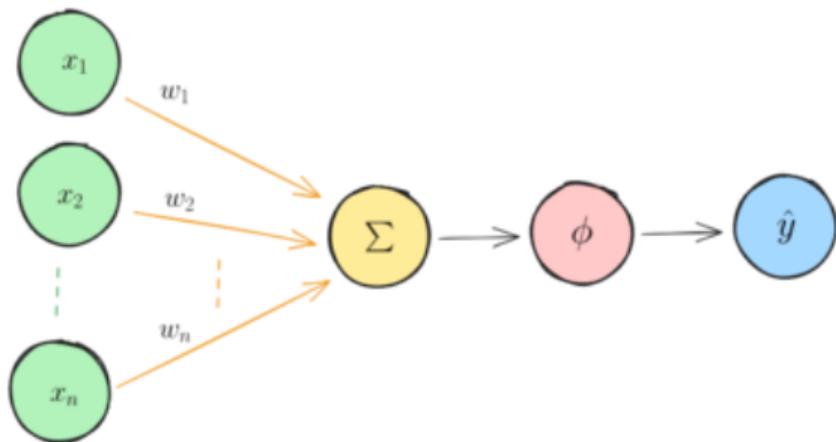
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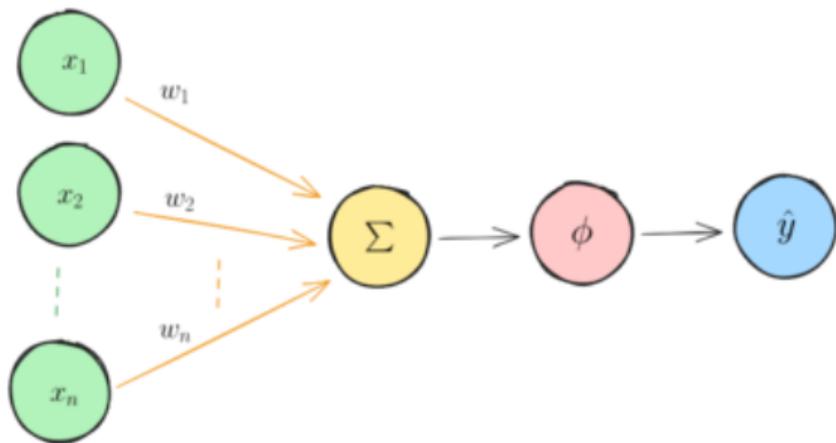
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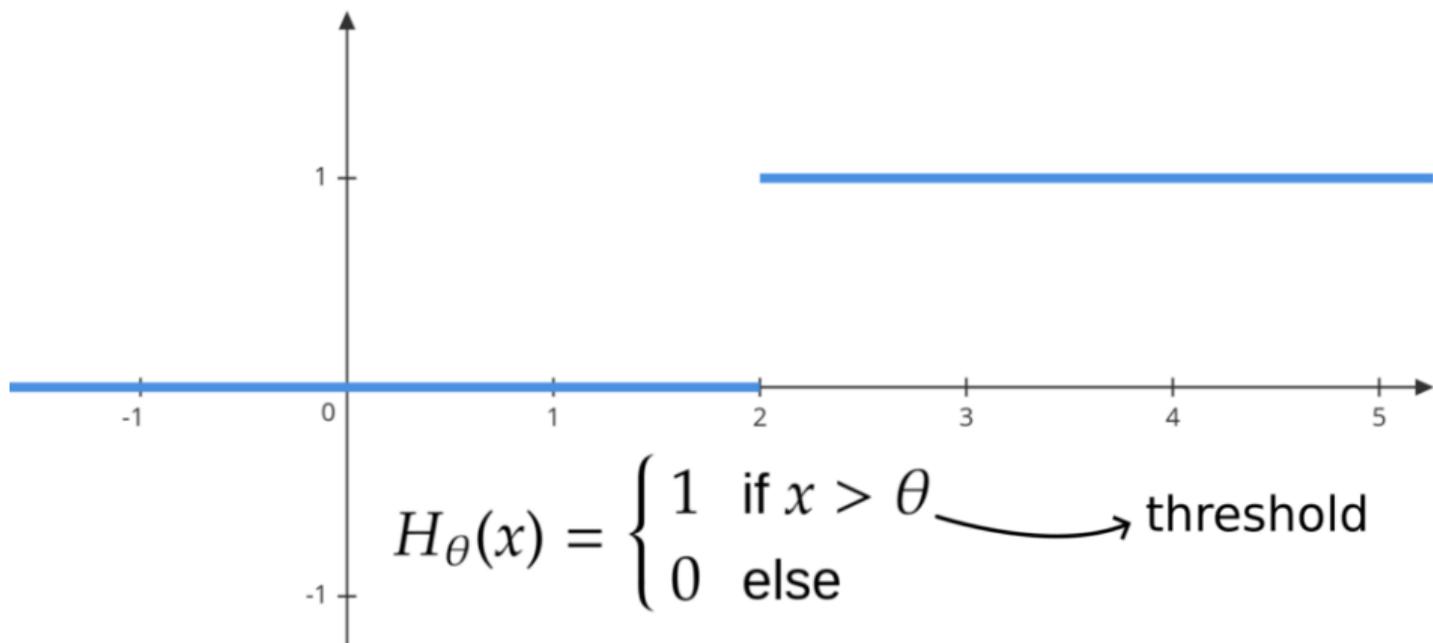
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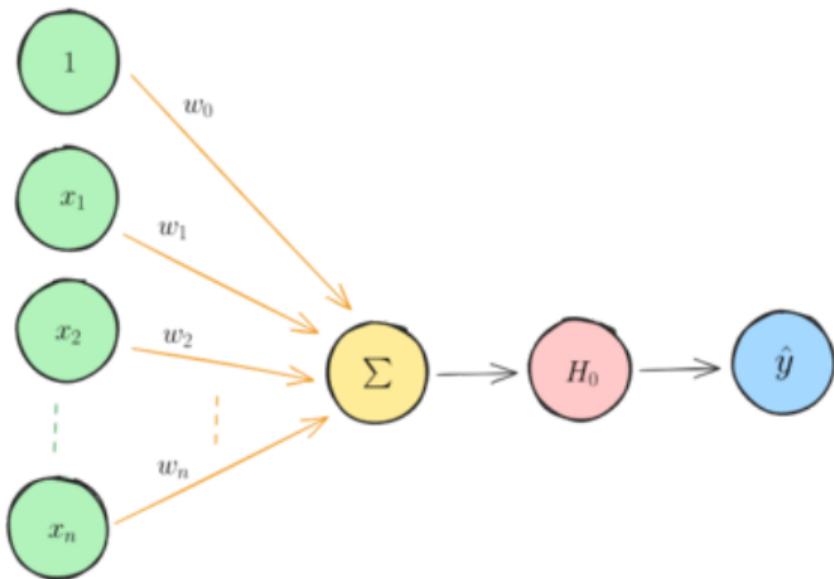
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# The artificial neuron [McCulloch and Pitts, 1943]



# The perceptron algorithm [Rosenblatt, 1957]



Using  $x_0 = 1$  and  $w_0 = -\theta$ :

$$\begin{aligned}\hat{y} &= H_{\theta}\left(\sum_{i=1}^n x_i w_i\right) \\ &= H_0\left(\sum_{i=0}^n x_i w_i\right) \\ &= H_0(\mathbf{x}^T \mathbf{w})\end{aligned}$$

$$\text{with } \mathbf{x} = \begin{bmatrix} x_0 \\ \vdots \\ x_n \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} w_0 \\ \vdots \\ w_n \end{bmatrix}$$

# The perceptron: learning

Instead of using hand-set values for weights, Rosenblatt proposes to **learn** them.

$$\text{Learning rule: } \Delta w_i = \eta x_i (y - \hat{y})$$

→ Intuitively, if the prediction is larger than the target, we need to reduce the weights, and vice versa.

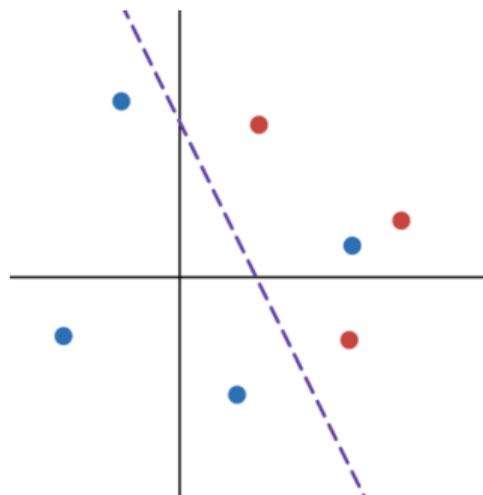
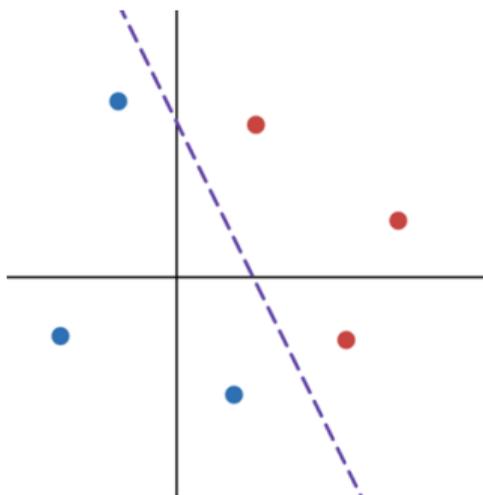
Let's learn the **OR** function by iterating on four learning examples:

$$\mathbf{x}^1 = \begin{bmatrix} \mathbf{1} \\ 0 \\ 0 \end{bmatrix} \rightarrow 0, \quad \mathbf{x}^2 = \begin{bmatrix} \mathbf{1} \\ 1 \\ 0 \end{bmatrix} \rightarrow 1, \quad \mathbf{x}^3 = \begin{bmatrix} \mathbf{1} \\ 0 \\ 1 \end{bmatrix} \rightarrow 1, \quad \mathbf{x}^4 = \begin{bmatrix} \mathbf{1} \\ 1 \\ 1 \end{bmatrix} \rightarrow 1$$

# The perceptron: properties

## Properties

1. linear classifier, i.e., separates space with an **hyperplan**
2. weight vector is orthogonal to the hyperplan, bias controls the y-intercept
3. converges for infinitesimally small  $\eta$  if the training data is linearly separable



# From perceptron to SGD

## Problems with the perceptron:

- can only perform binary classification
- does not converge when data is not linearly separable (or noisy)
- updates in an abrupt manner and does not use well classified samples

## Stochastic gradient descent (SGD):

**Goal:** update weights to minimize the cost function  $\mathcal{J}$

$$\Delta \mathbf{w} = -\eta \nabla \mathcal{J}(\mathbf{w})$$

- updates in a smoother way than perceptron (uses all samples)
- converges even for non linearly separable data (for appropriately chosen  $\eta$ )
- needs a differentiable cost function!

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# Gradient descents

---

## Algorithm 1: Gradient descent

---

**Data:** Training dataset of  $N$  examples

**Result:** Optimized weights  $\mathbf{w}$

**Initialize** weights randomly;

**while** *not converged* **do**

    Compute true gradient,  $\nabla J(\mathbf{w}) = \frac{1}{N} \sum_1^N \mathcal{L}(x_i)$  // Expensive but convergence is theoretically guaranteed

    Update weights,  $\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla J(\mathbf{w})$ ;

**end**

**return**  $\mathbf{w}$ ;

---

- Batch GD:  $\mathcal{J}$  is the average of a loss  $\mathcal{L}$  over the entire dataset
- Online GD:  $\mathcal{J}$  is the loss on a single training example
- Mini-batch GD:  $\mathcal{J}$  is the average loss over a subset of the training dataset

Gradient descent algorithms are **stochastic** when the training examples are selected randomly.

# Gradient descents

---

**Algorithm 2:** Online gradient descent

---

**Data:** Training dataset of  $N$  examples

**Result:** Optimized weights  $\mathbf{w}$

**Initialize** weights randomly;

**while** *not converged* **do**

    Compute estimate gradient,  $\nabla J(\mathbf{w}) \simeq \mathcal{L}(x_i)$  // Faster, but noisier: one example  
    is not representative of the training data

    Update weights,  $\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla J(\mathbf{w})$ ;

**end**

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---

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# Gradient descents

---

## Algorithm 3: Mini-batch gradient descent

---

**Data:** Training dataset of  $N$  examples

**Result:** Optimized weights  $\mathbf{w}$

**Initialize** weights randomly;

**while** *not converged* **do**

    Compute estimate gradient,  $\nabla J(\mathbf{w}) \simeq \frac{1}{n} \sum_1^{n < N} \mathcal{L}(x_i)$  // Often best balance in practice

    Update weights,  $\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla J(\mathbf{w})$ ;

**end**

**return**  $\mathbf{w}$ ;

---

- Batch GD:  $\mathcal{J}$  is the average of a loss  $\mathcal{L}$  over the entire dataset
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Gradient descent algorithms are **stochastic** when the training examples are selected randomly.

# Gradient descents

---

**Algorithm 4:** Mini-batch gradient descent

---

**Data:** Training dataset of  $N$  examples

**Result:** Optimized weights  $\mathbf{w}$

**Initialize** weights randomly;

**while** *not converged* **do**

    Compute estimate gradient,  $\nabla J(\mathbf{w}) \simeq \frac{1}{n} \sum_{1 \leq i < N} \mathcal{L}(x_i)$  // Often best balance in practice

    Update weights,  $\mathbf{w} \leftarrow \mathbf{w} - \eta \nabla J(\mathbf{w})$ ;

**end**

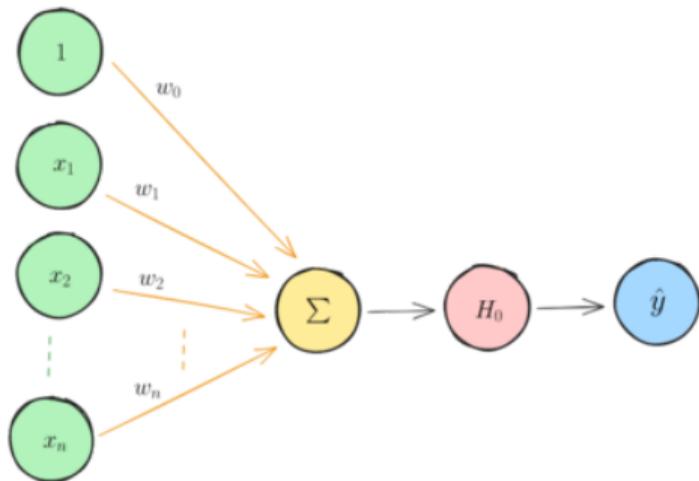
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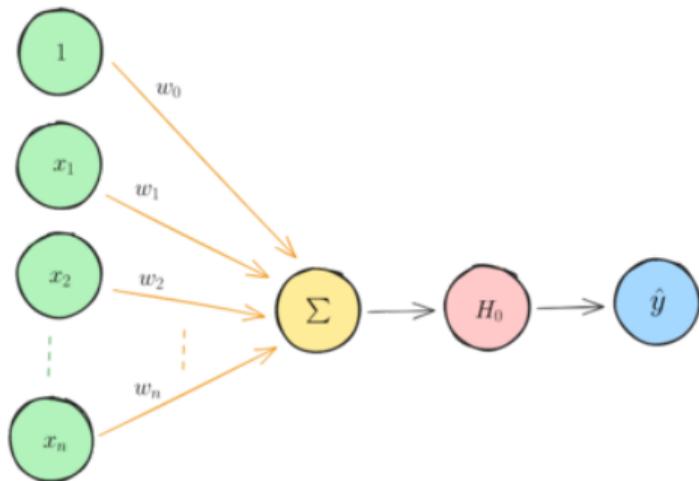
Gradient descent algorithms are **stochastic** when the training examples are selected randomly.

# SGD example: house price prediction



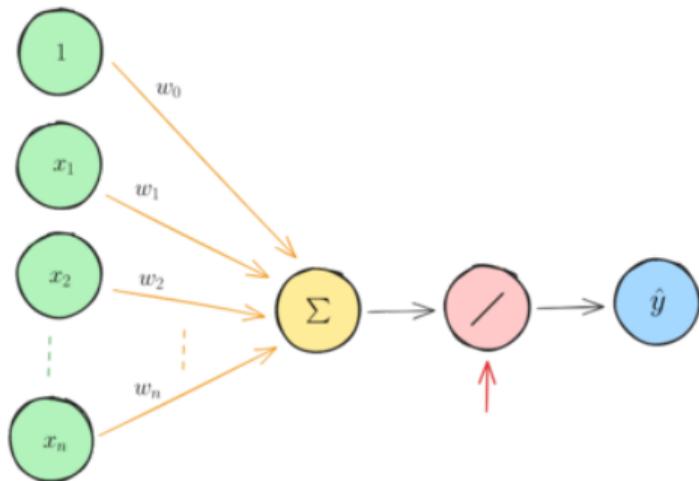
- features  $x \in \{\text{surface area, number of rooms, exposure, parking...}\}$
- labels  $y \in \mathbb{R}$   
→ need to change activation!  
 $\phi(x) = x$  is simple, differentiable, and its codomain is  $\mathbb{R}$
- What cost function should we use?  
let's try the average error  
 $\frac{1}{n} \sum_x y - \hat{y}(x)$

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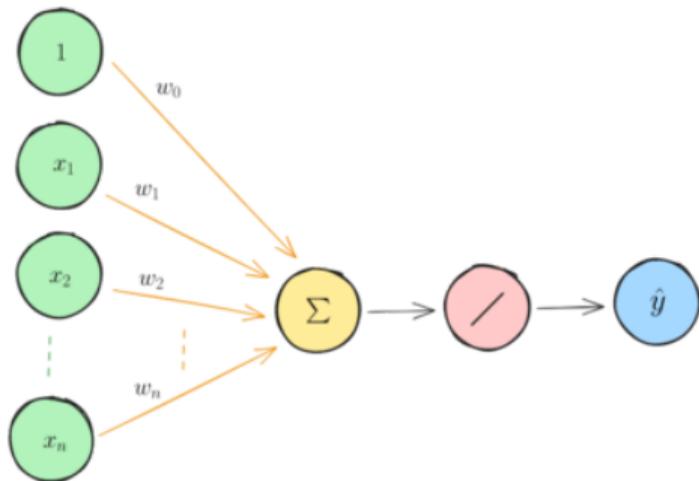
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- What cost function should we use?  
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 $\frac{1}{n} \sum_x y - \hat{y}(x)$

## SGD: mean error

$\Delta \mathbf{w} = -\eta \nabla \mathcal{J}(\mathbf{w})$ , and specifically:

$$\begin{aligned}\Delta w_i &= -\eta \frac{\partial}{\partial w_i} \mathcal{J}(\mathbf{w}) \\ &= -\eta \frac{\partial}{\partial w_i} \frac{1}{n} \sum_x y - \hat{y}(x) \\ &= -\eta \frac{1}{n} \sum_x \frac{\partial}{\partial w_i} y - \hat{y}(x) \\ &= -\eta \frac{1}{n} \sum_x \frac{\partial}{\partial w_i} y - \sum_i x_i w_i \\ &= \eta \frac{1}{n} \sum_x x_i\end{aligned}$$

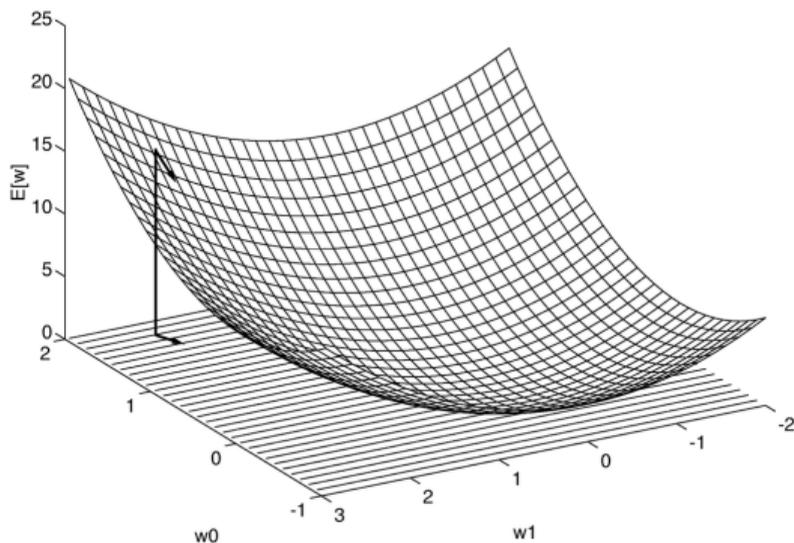
No dependence on the target! The weights will drift without ever converging.

$-10 + 10 = 0 \rightarrow$  loss should be non-negative!

# SGD: mean squared error (MSE)

$$\text{MSE} = \frac{1}{2n} \sum_x (y - \hat{y}(x))^2$$

$$\begin{aligned}\Delta w_i &= -\eta \frac{\partial}{\partial w_i} \mathcal{J}(\mathbf{w}) \\ &= -\eta \frac{\partial}{\partial w_i} \frac{1}{2n} \sum_x (y - \hat{y}(x))^2 \\ &= \frac{-\eta}{2n} \sum_x \frac{\partial}{\partial w_i} (y - \hat{y}(x))^2 \\ &= \frac{-\eta}{2n} \sum_x -2x_i (y - \hat{y}(x)) \\ &= \frac{\eta}{n} \sum_x (y - \hat{y}(x)) x_i\end{aligned}$$



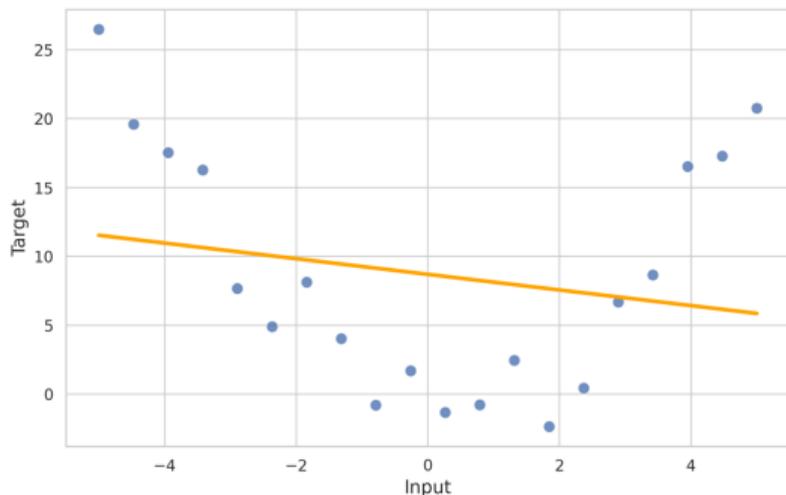
The choice of loss function is important!

# Beyond learning linear functions

We are learning weights for a perceptron:  
a linear combination of inputs.

**How can we learn non-linear functions?**

Use multiple layers of neurons!



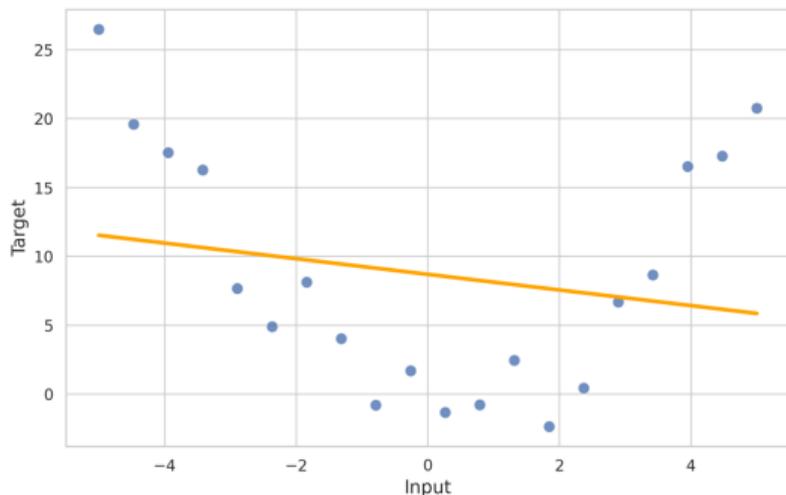
Our perceptron learns the linear best fit, but we can do better.

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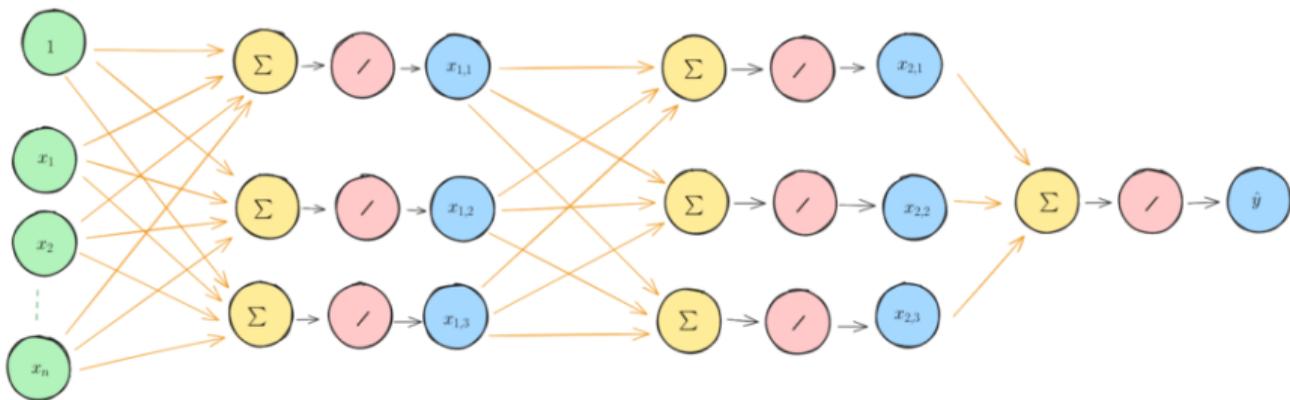
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Our perceptron learns the linear best fit, but we can do better.

# Multi-layer perceptron (MLP)



**neural network:** a series of layers with weights and activations, transforming an input into an output.

Can this learn non-linear function? Let's put it to the [test!](#)

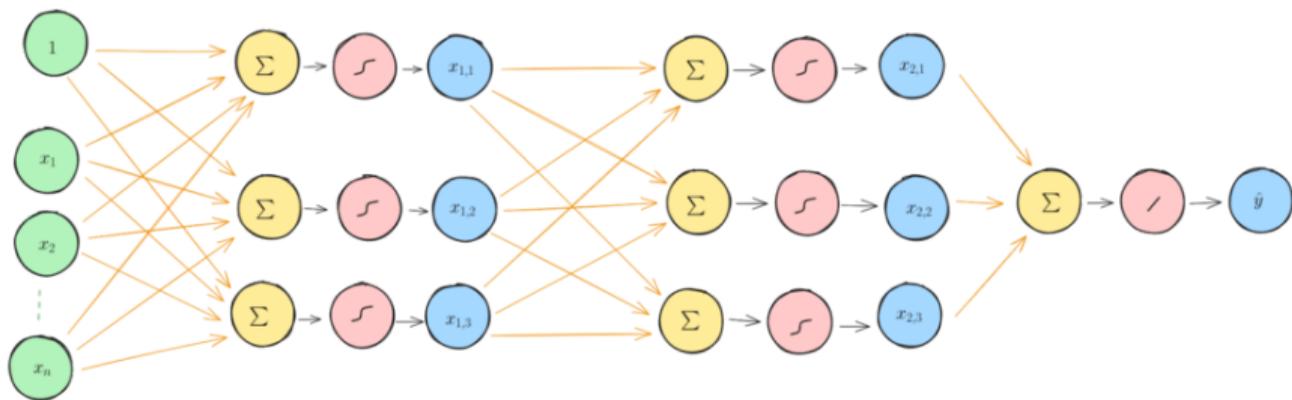
# Why Non-Linear Activations?

What happens if we stack multiple linear layers?

$$\begin{aligned}\hat{\mathbf{y}} &= \mathbf{W}^{(3)} \mathbf{a}^{(2)} + \mathbf{b}^{(3)} \\ &= \mathbf{W}^{(3)} (\mathbf{W}^{(2)} \mathbf{a}^{(1)} + \mathbf{b}^{(2)}) + \mathbf{b}^{(3)} \\ &= \mathbf{W}^{(3)} (\mathbf{W}^{(2)} (\mathbf{W}^{(1)} \mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)}) + \mathbf{b}^{(3)} \\ &= \mathbf{W}^{(3)} (\mathbf{W}^{(2)} \mathbf{W}^{(1)} \mathbf{x} + \mathbf{W}^{(2)} \mathbf{b}^{(1)} + \mathbf{b}^{(2)}) + \mathbf{b}^{(3)} \mathbf{W}^{(2)} \\ &= (\mathbf{W}^{(3)} \mathbf{W}^{(2)} \mathbf{W}^{(1)}) \mathbf{x} + (\mathbf{W}^{(3)} \mathbf{W}^{(2)} \mathbf{b}^{(1)} + \mathbf{W}^{(3)} \mathbf{b}^{(2)} + \mathbf{b}^{(3)}) \\ &= \mathbf{W}' \mathbf{x} + \mathbf{b}'\end{aligned}$$

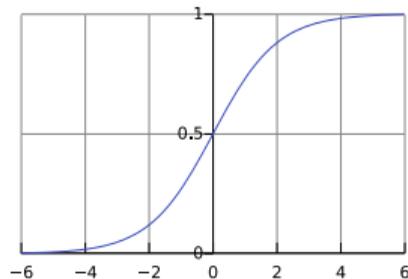
A deep network of linear layers can only represent linear functions!

# Multi-layer perceptron (MLP)



We need to introduce non-linearities, e.g., using the sigmoid as the activation functions in hidden layers.

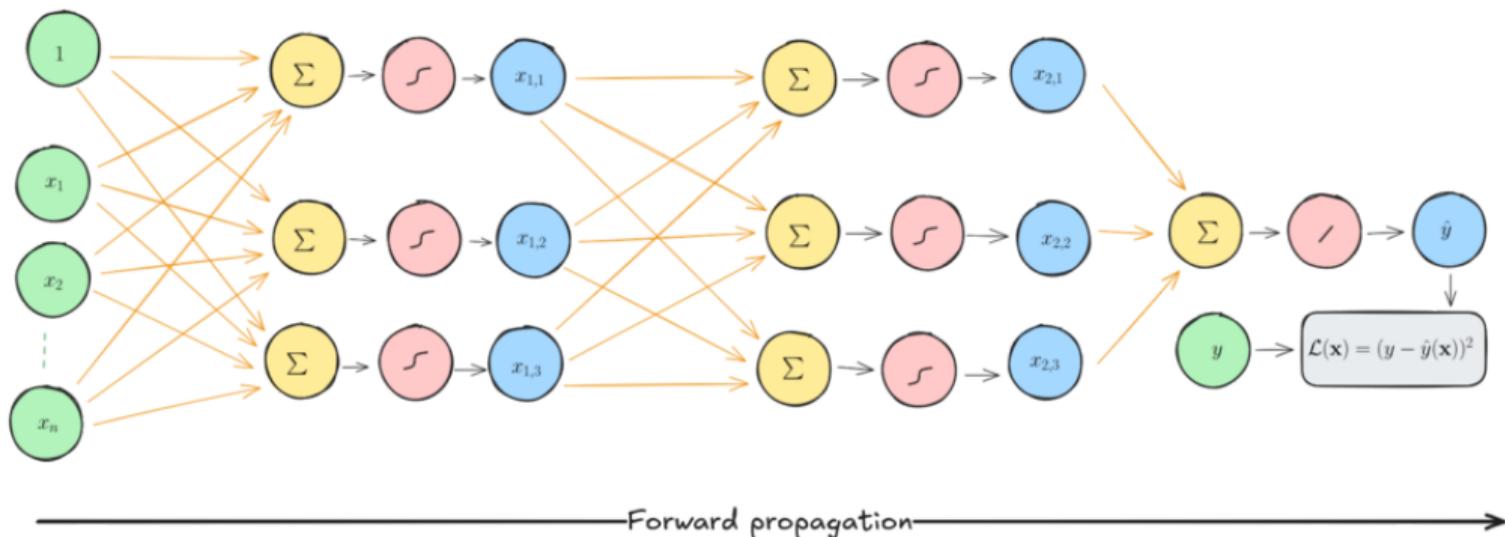
$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad \sigma'(x) = (1 - \sigma(x))\sigma(x)$$



# Learning with a MLP

Two phases:

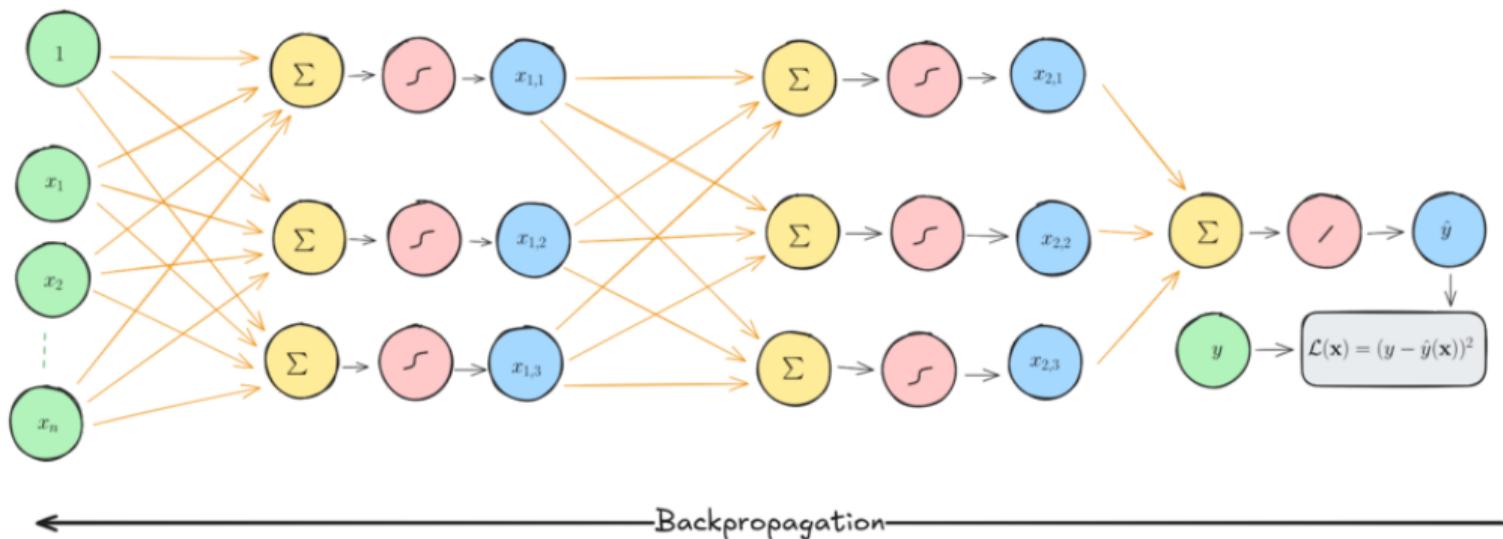
- **forward propagation** (inference)  
input passes through the network to produce the output, used to compute the loss
- **backpropagation**



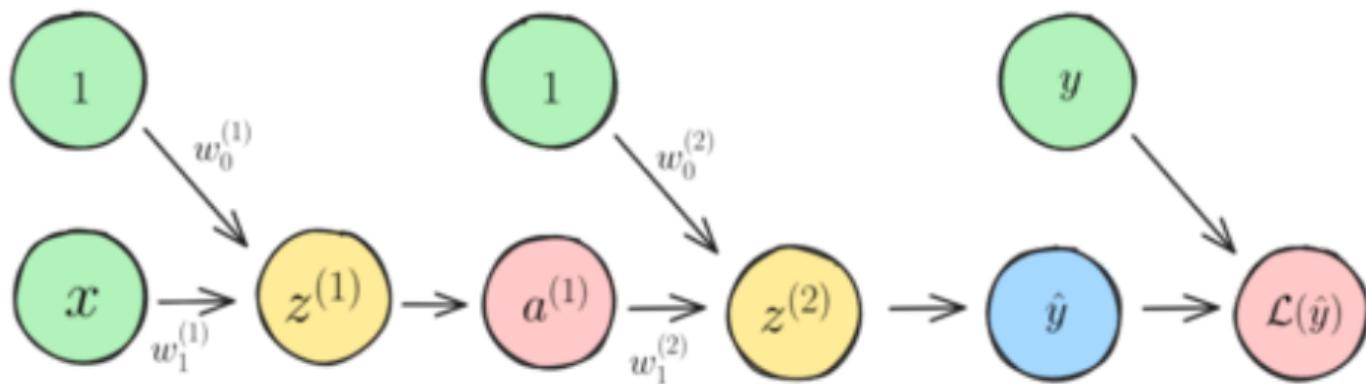
# Learning with a MLP

Two phases:

- forward propagation (inference)
- **backpropagation**  
gradients are propagated backward through the network, allowing us to perform an SGD update



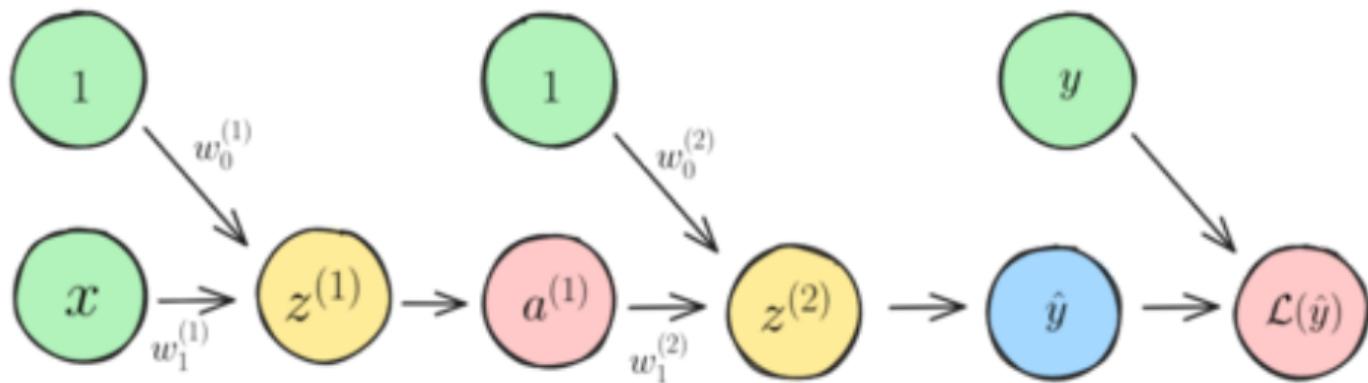
## Backpropagation: simple example



What is the influence of  $w_1^{(1)}$  on  $\mathcal{L}(\hat{y})$ ?  
How should I modify its value to decrease the loss?

$$\frac{\partial \mathcal{L}}{\partial w_1^{(1)}} = ?$$

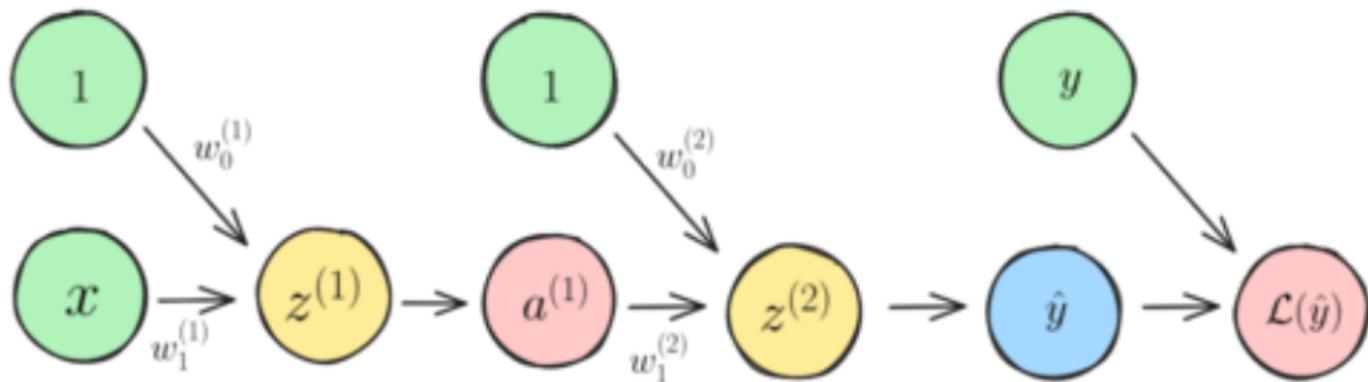
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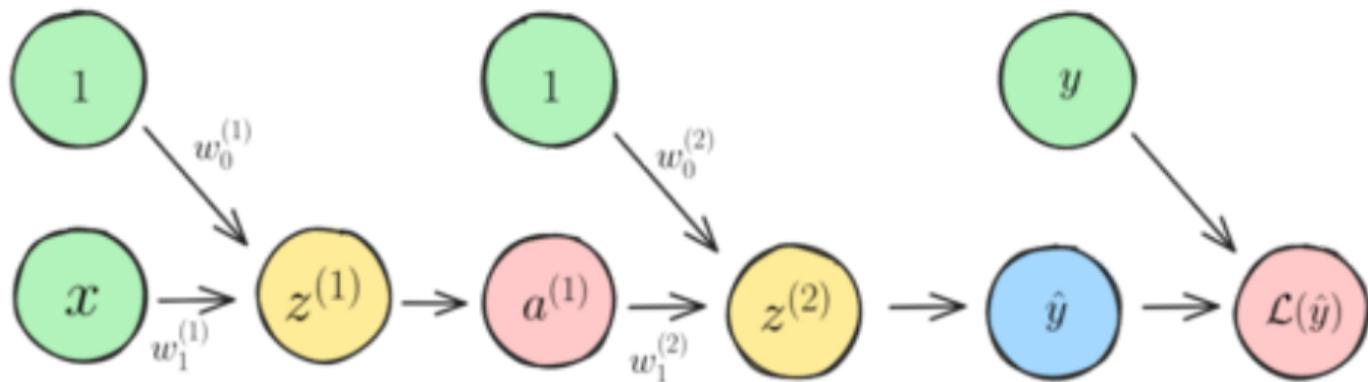
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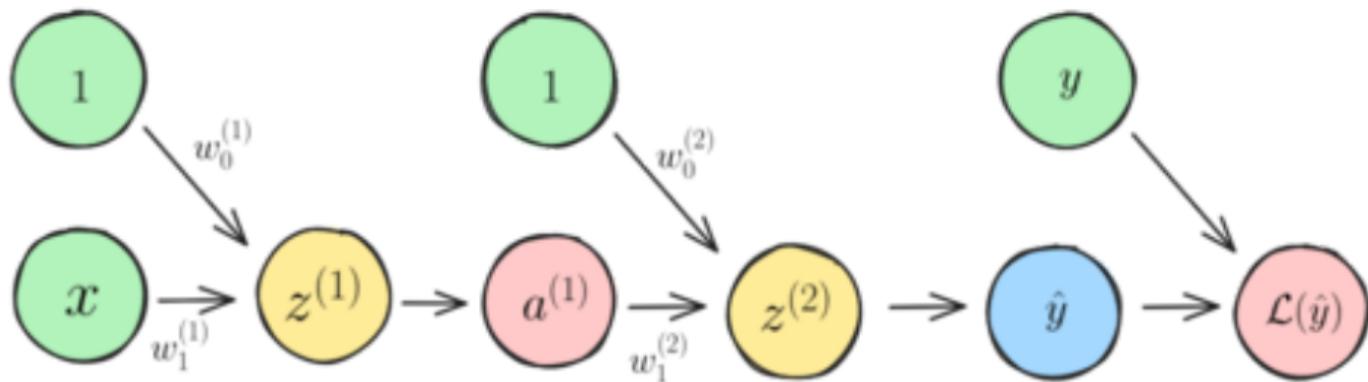
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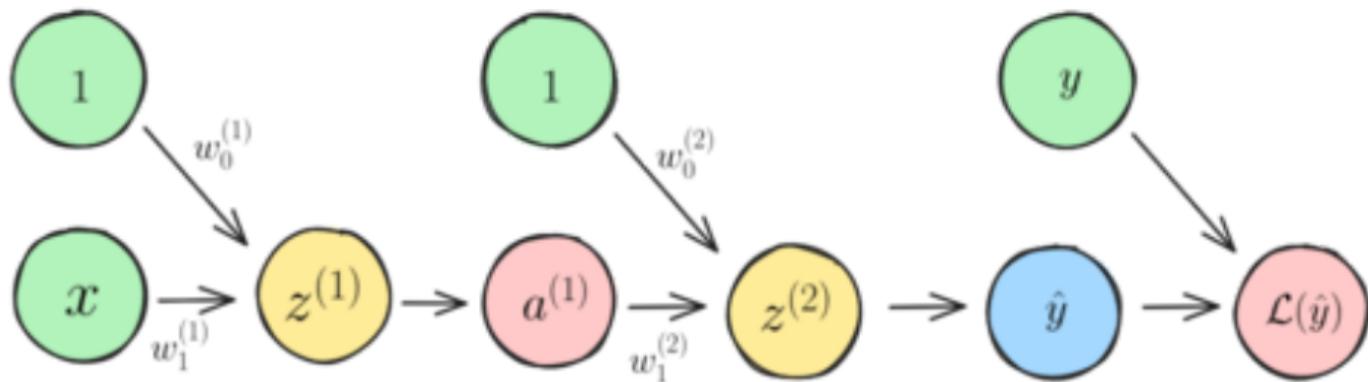
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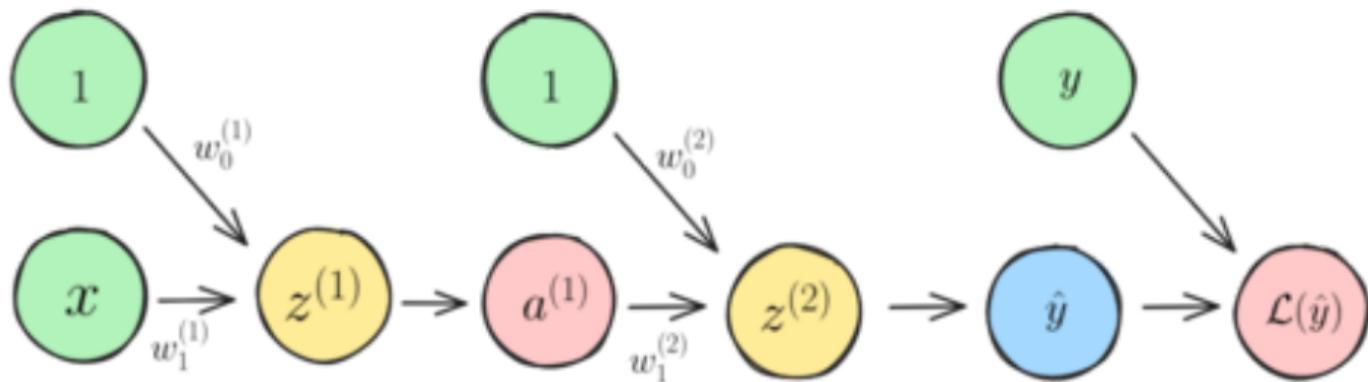
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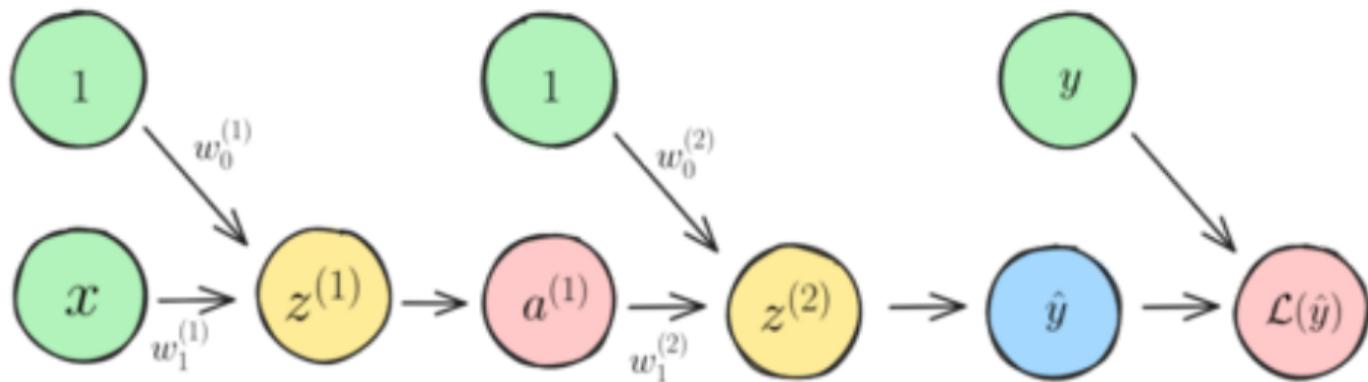
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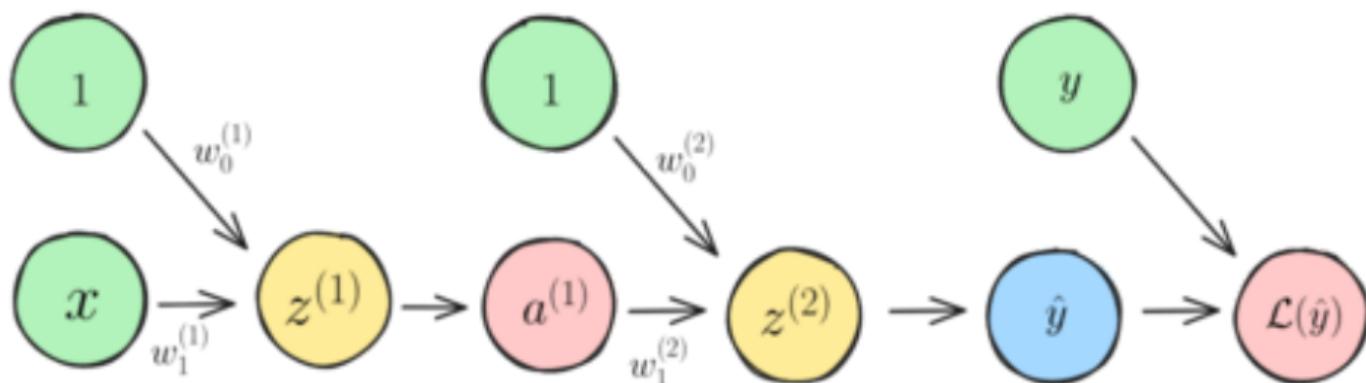
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## Backpropagation: simple example



Remembering SGD,  $\Delta w = -\eta \nabla \mathcal{J}(w)$

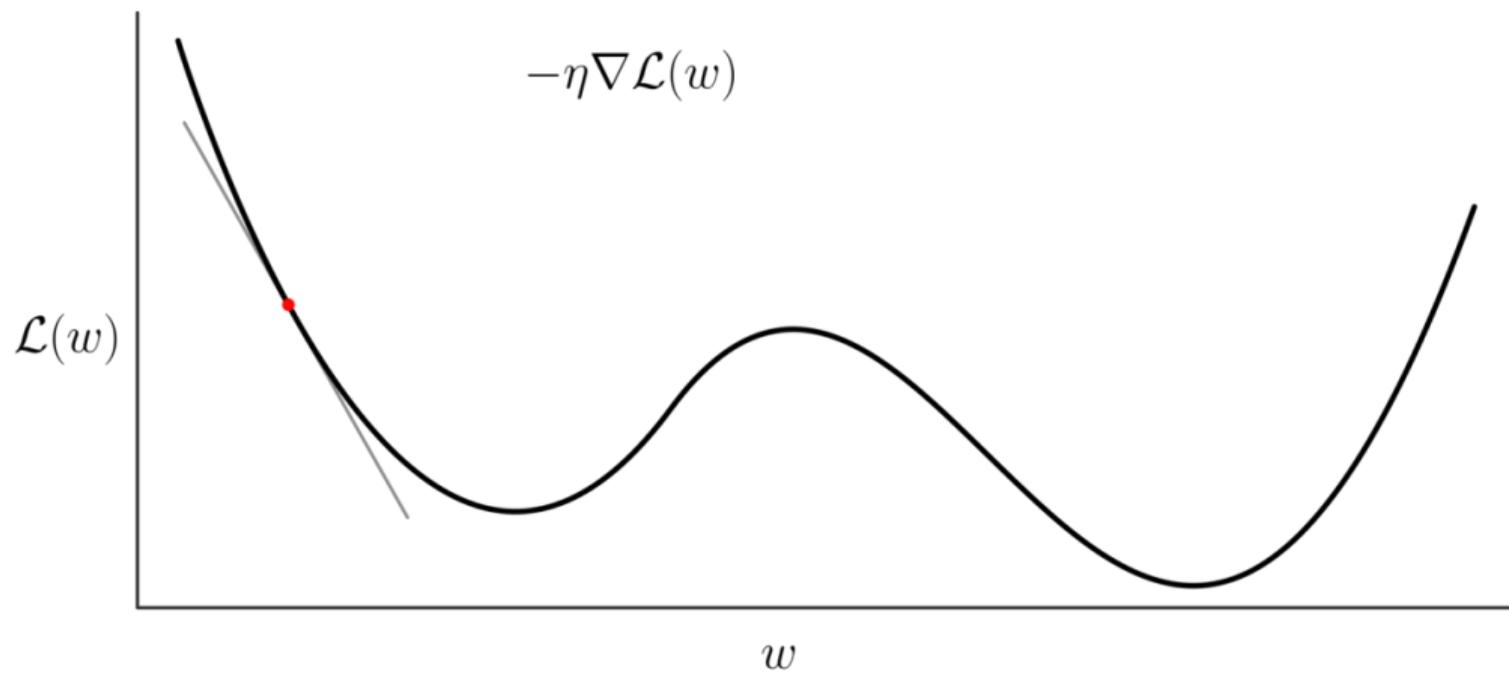
Hence,  $\Delta w_1^{(1)} = \eta 2(y - \hat{y})w_1^{(2)}\sigma(z^{(1)})(1 - \sigma(z^{(1)}))x$

We can reuse computations:

$$\Delta w_0^{(1)} = \eta 2(y - \hat{y})w_1^{(2)}\sigma(z^{(1)})(1 - \sigma(z^{(1)}))x$$

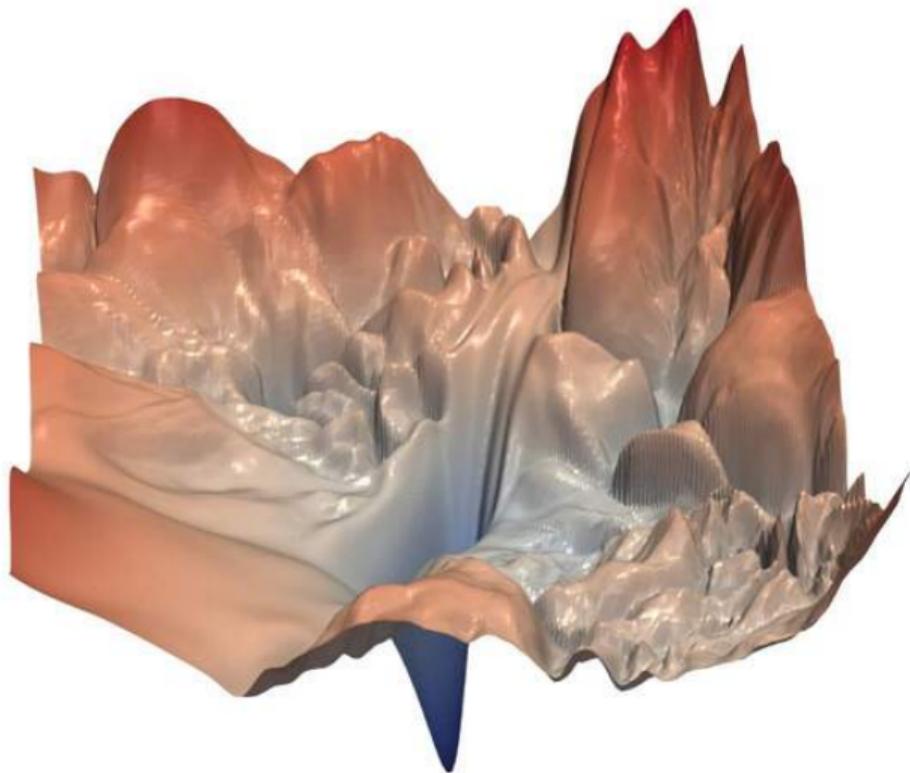
$$\Delta w_0^{(2)} = \eta 2(y - \hat{y})w_1^{(2)}\sigma(z^{(1)})(1 - \sigma(z^{(1)}))x$$

# Learning rate



# Finding the optimal weights

In practice, the loss landscape is very complex with billions of dimensions!



## MLP: summary

Multi-layer perceptrons are **universal approximators**: they can approximate any continuous function given that they are wide/deep enough.

**But** convergence can be ineffective (non-convex and high-dimensional space, vanishing gradients...) and may require some tricks in practice.

To play around with MLPs online: <https://playground.tensorflow.org>

# Convolutional Neural Networks (CNN) [LeCun et al., 1989]

How can we learn from images as inputs?

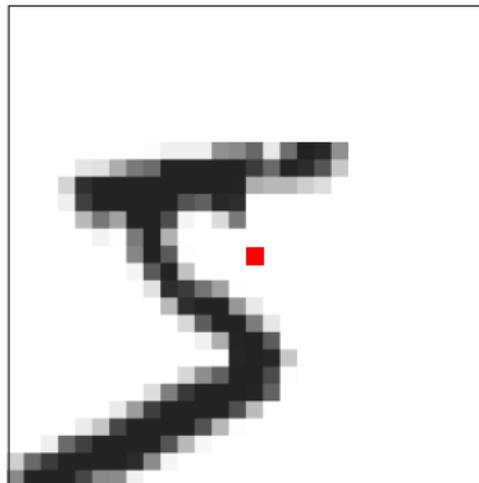
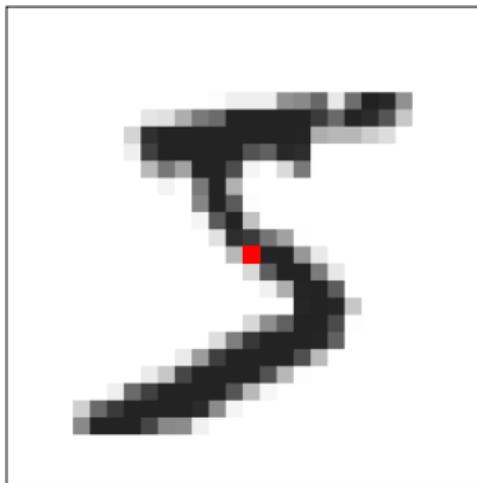
# Convolutional Neural Networks (CNN) [LeCun et al., 1989]

How can we learn from images as inputs?

## (Bad) solution

We can use an MLP! However:

- a huge number of weights to learn (an image has at least 1000 dimensions)
- the problem of translation

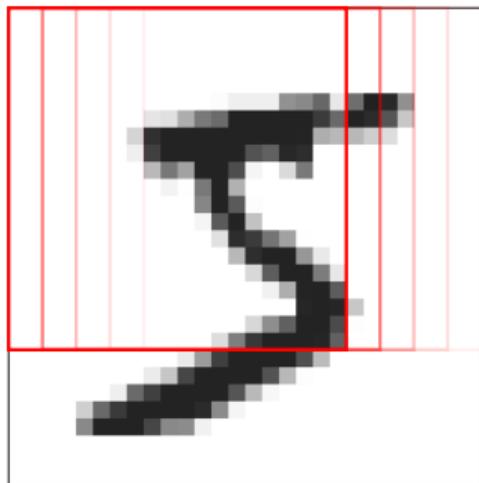


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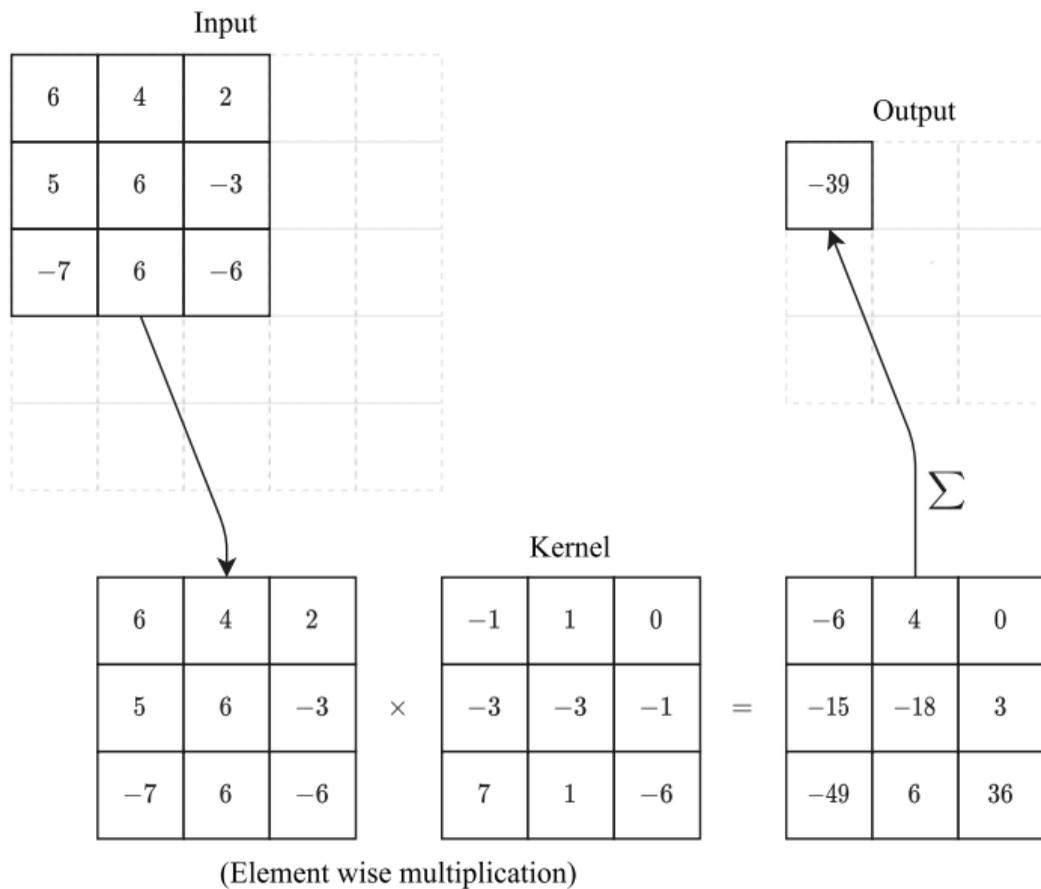
How can we learn from images as inputs?

## Good solution

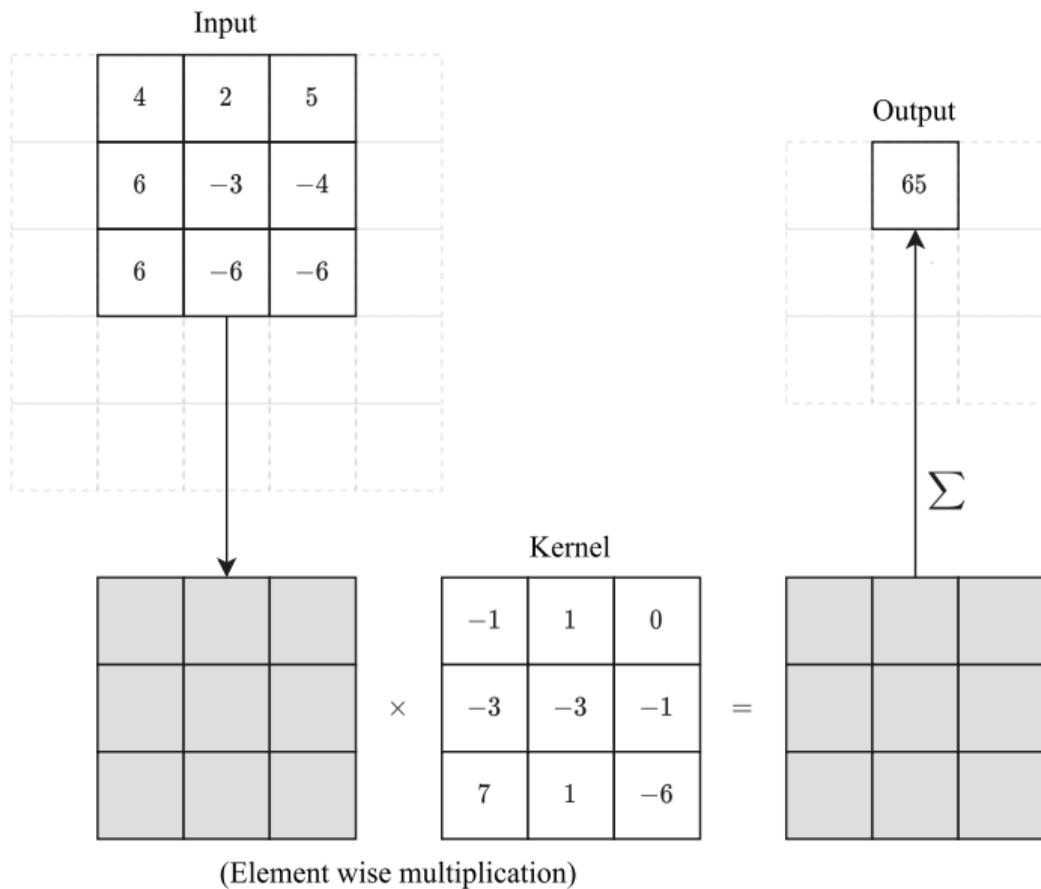
Adapt the neurons and network to perform convolution.



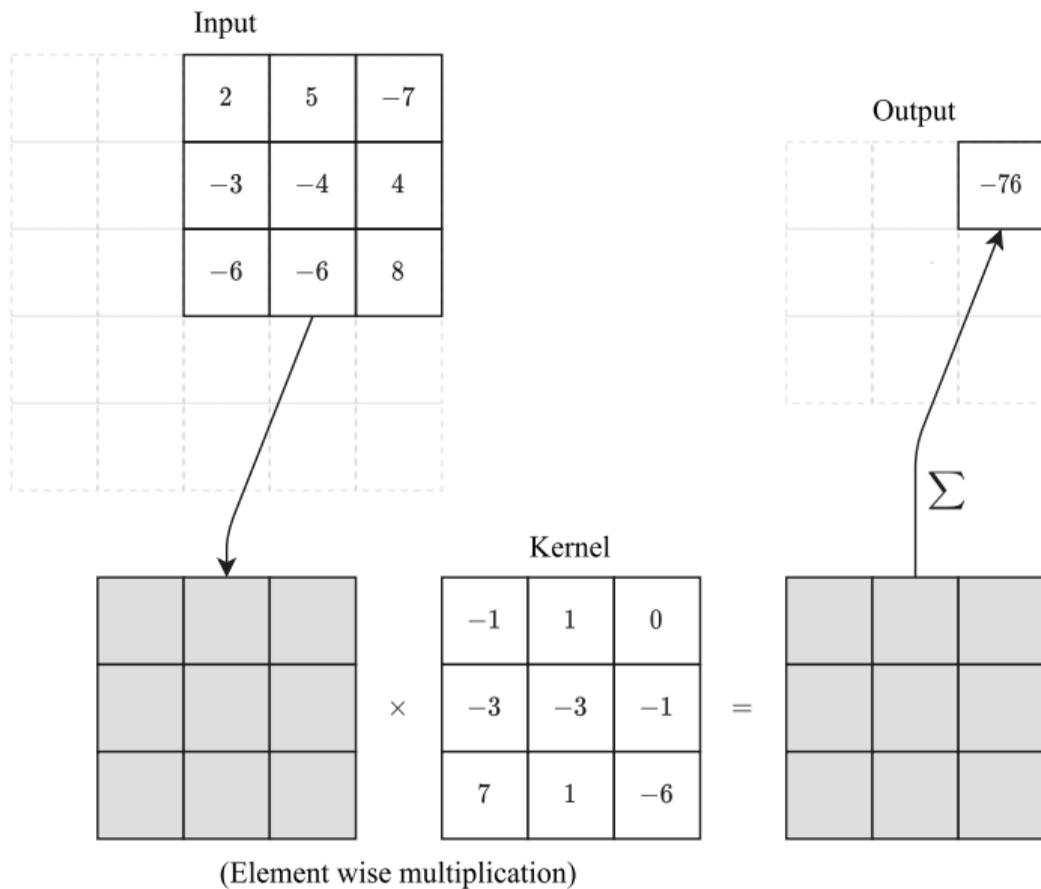
# CNN: the convolution operation



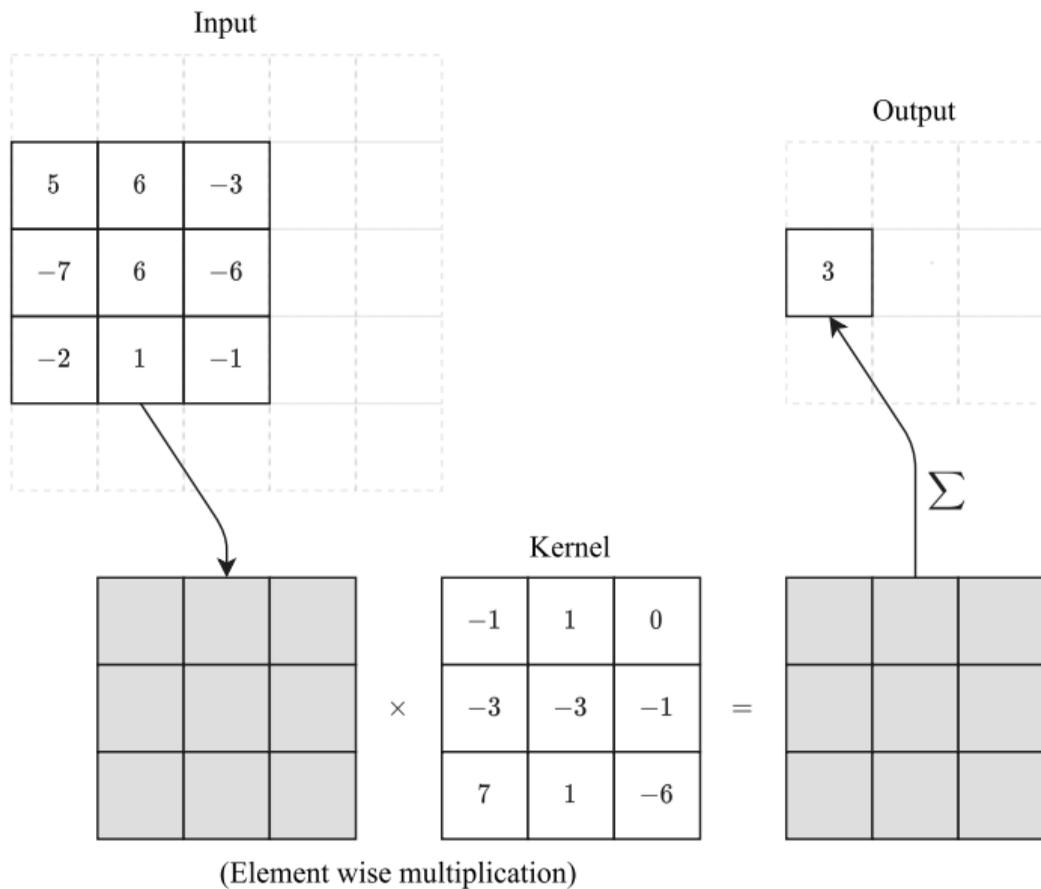
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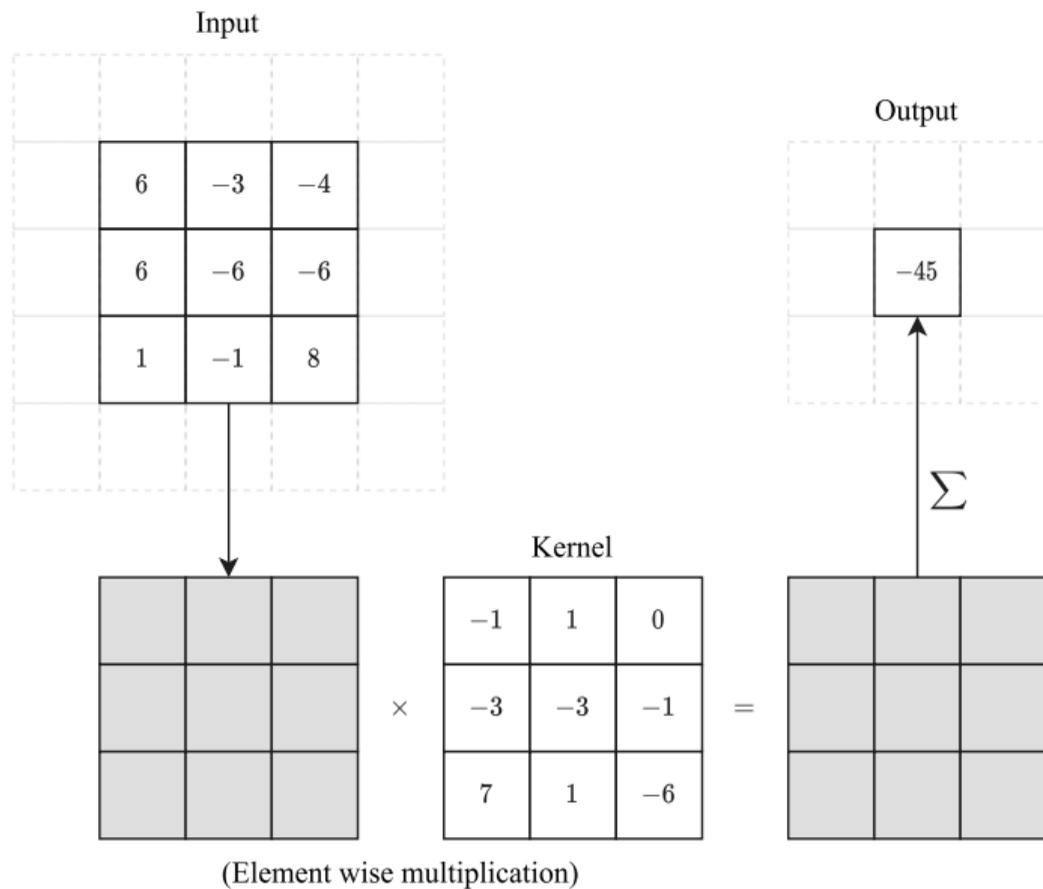
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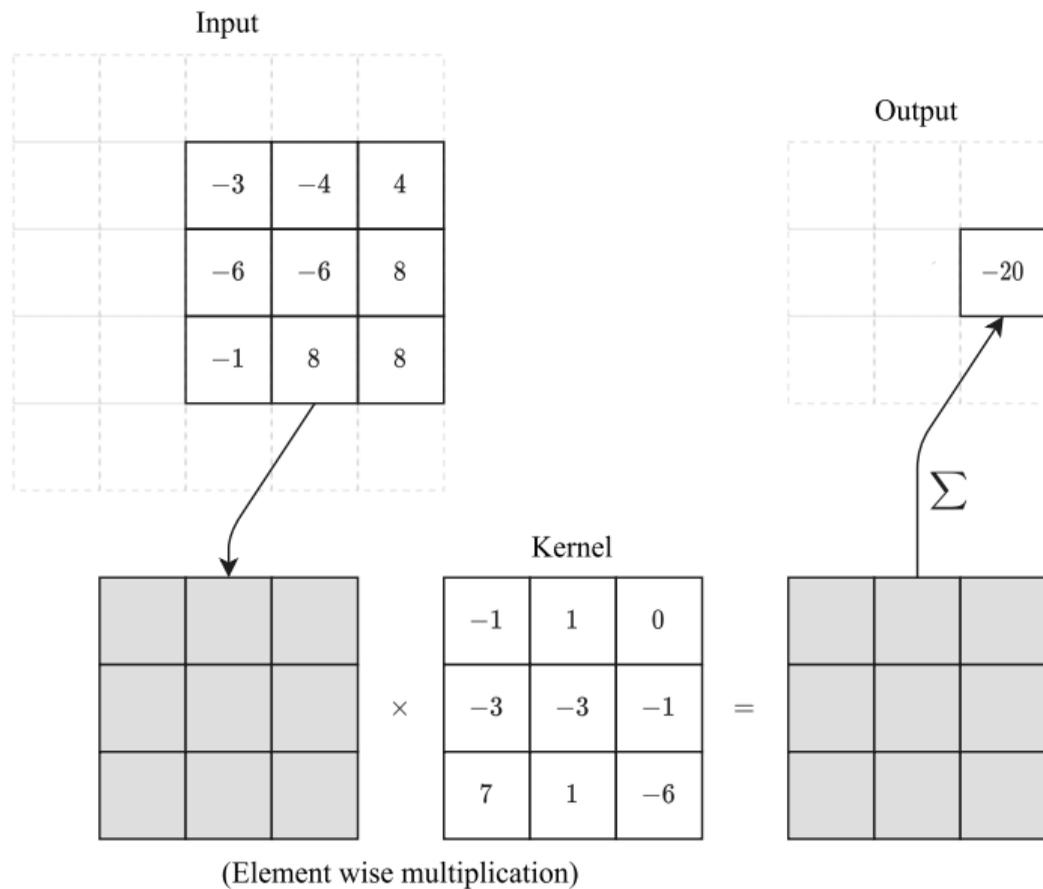
# CNN: the convolution operation



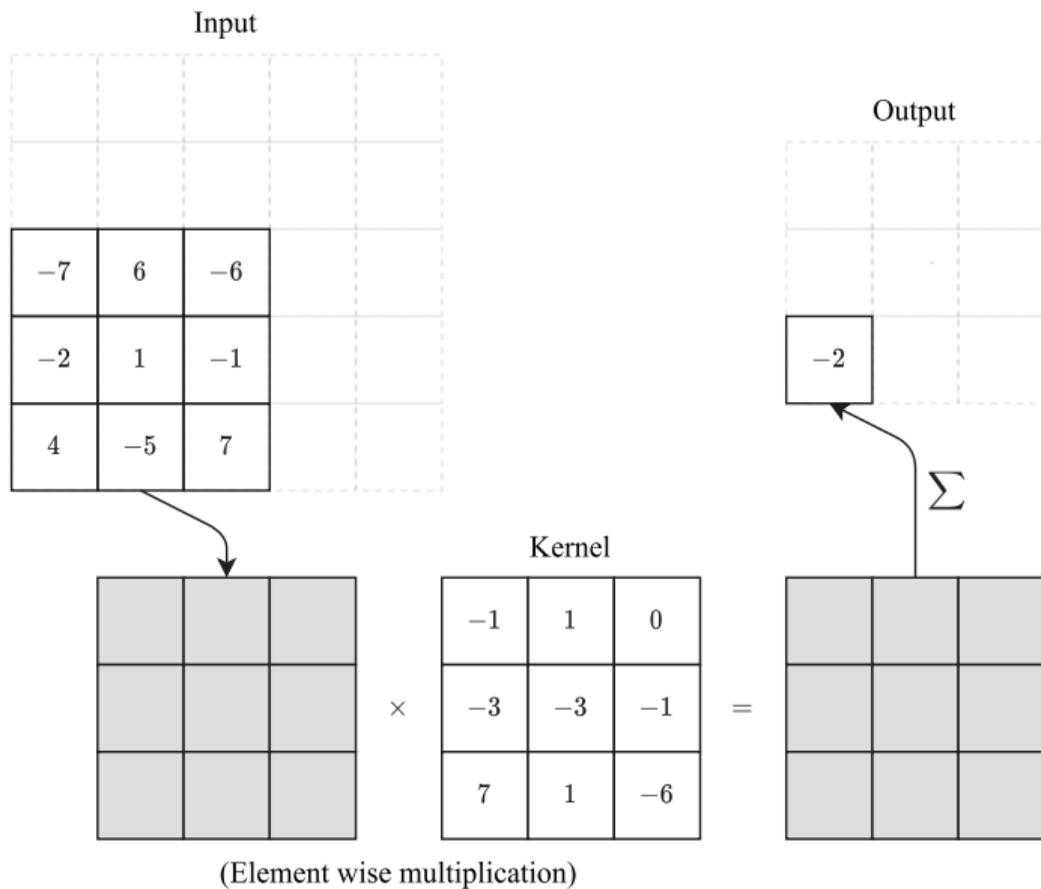
# CNN: the convolution operation



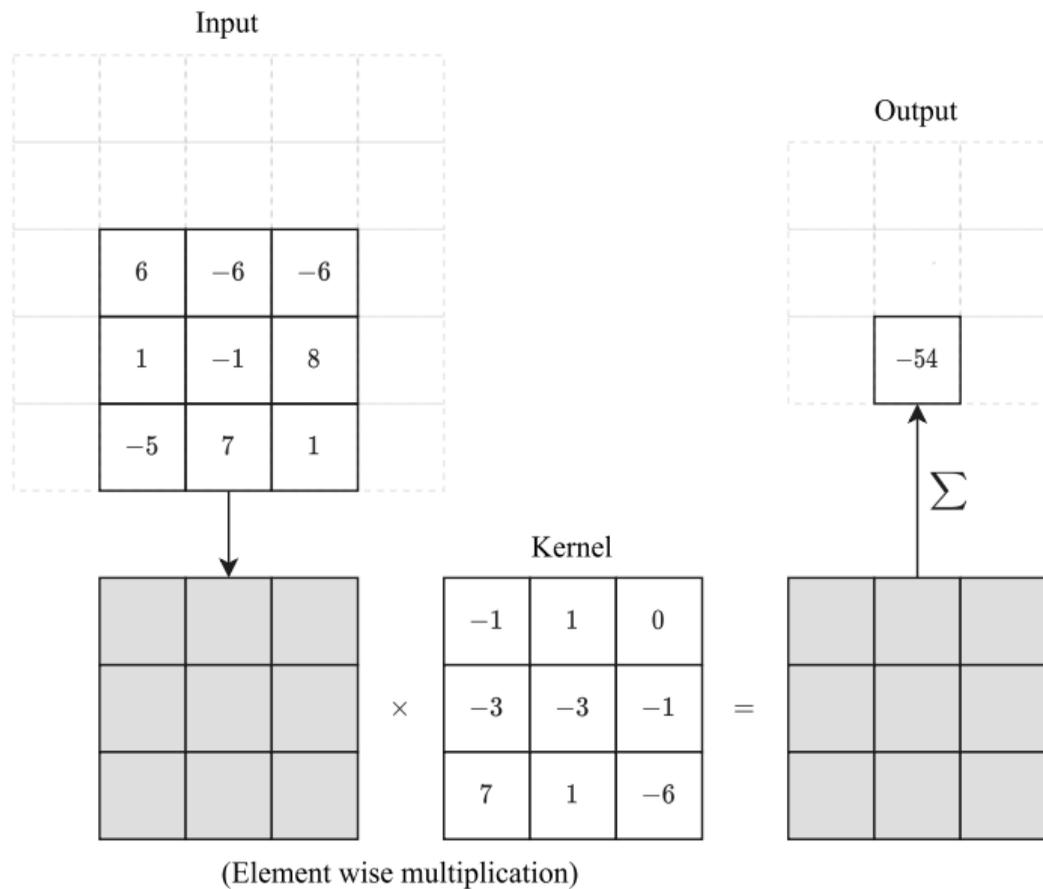
# CNN: the convolution operation



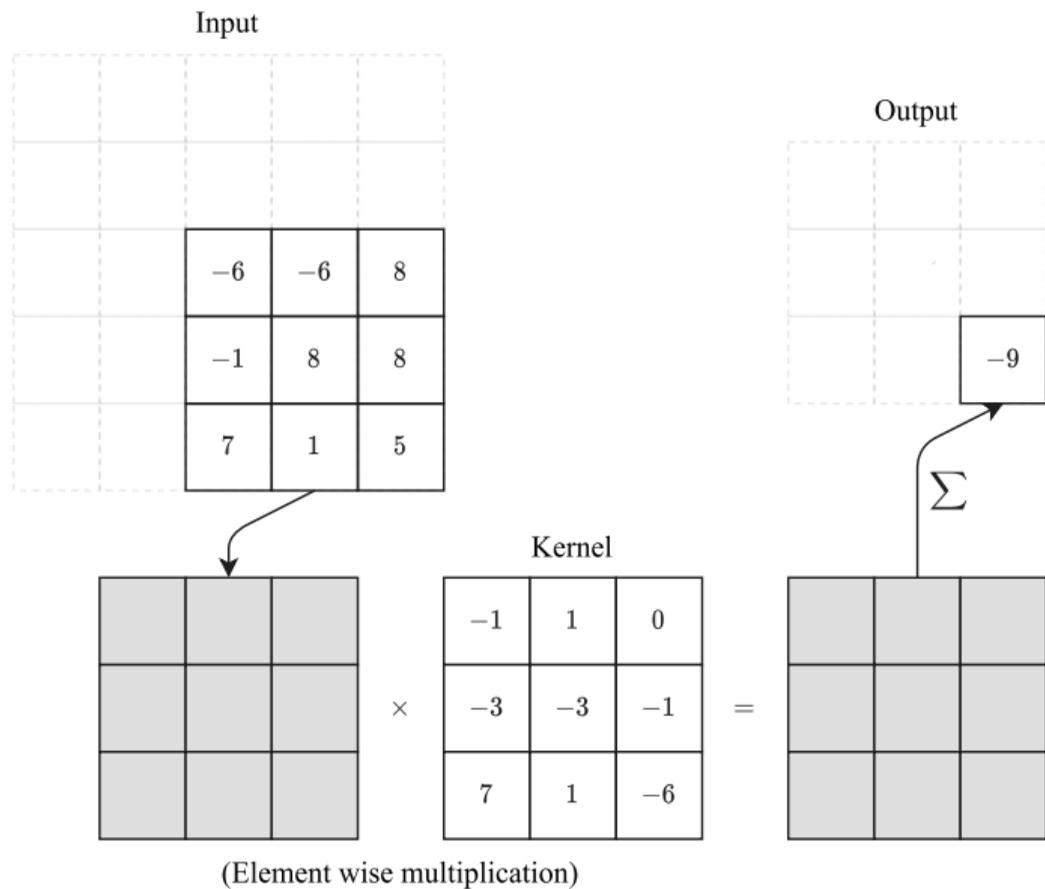
# CNN: the convolution operation



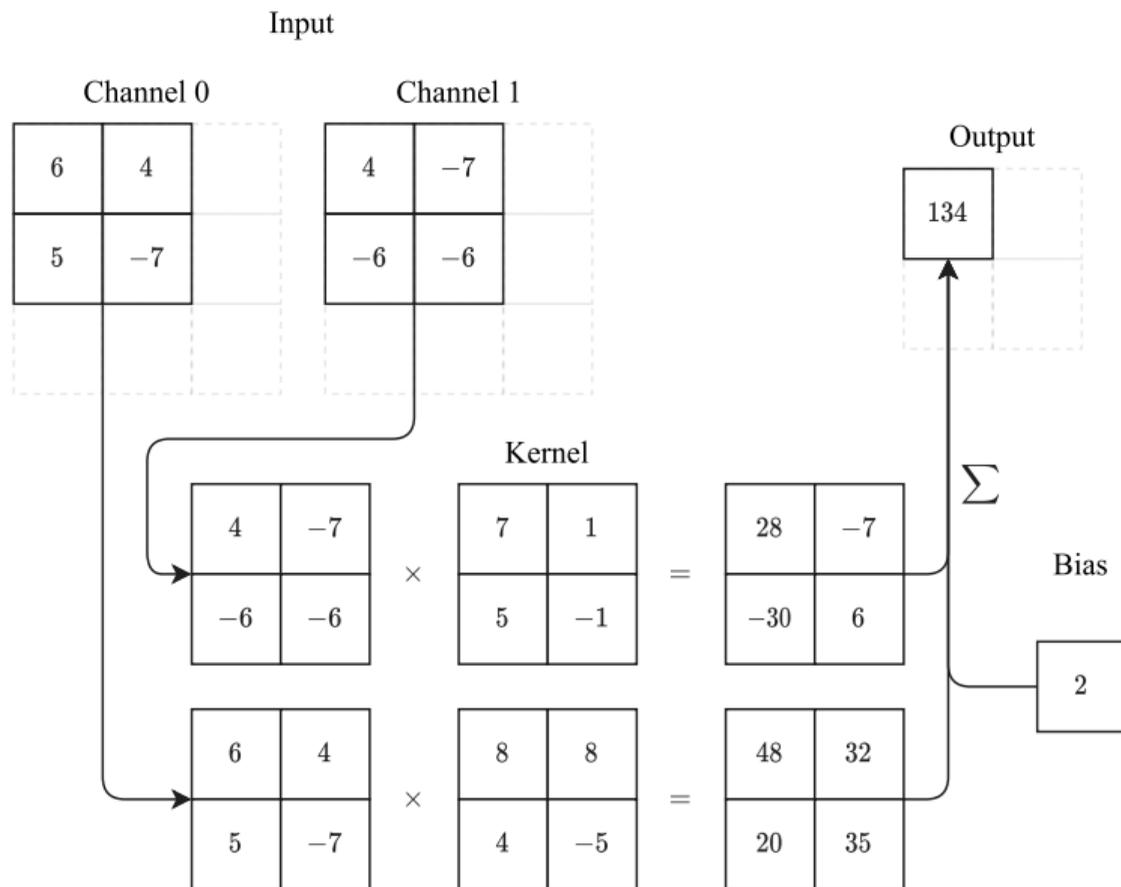
# CNN: the convolution operation



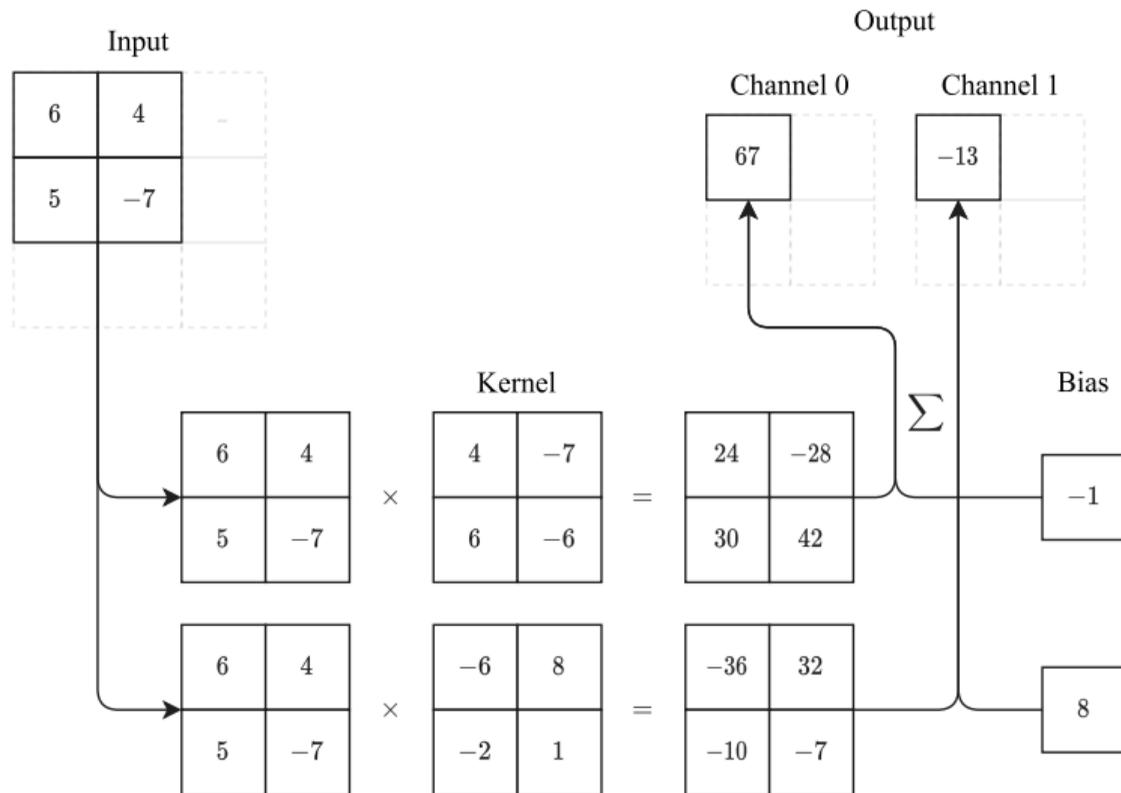
# CNN: the convolution operation



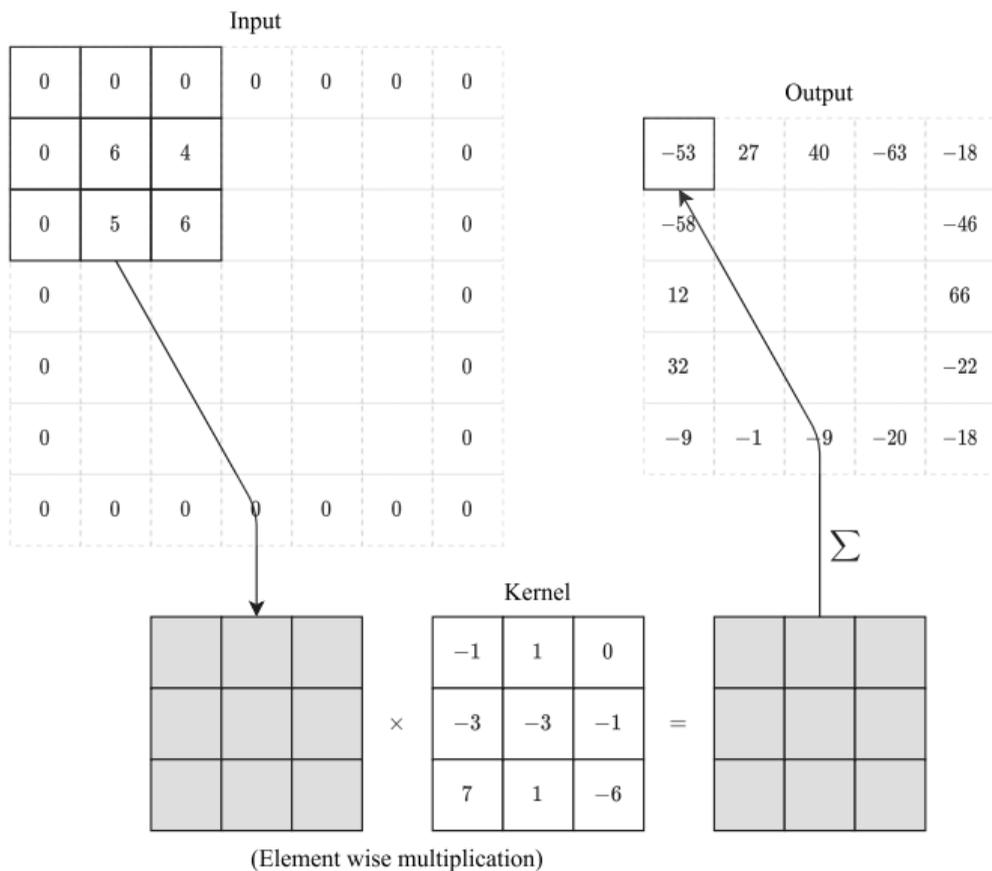
# CNN: convolutional neuron



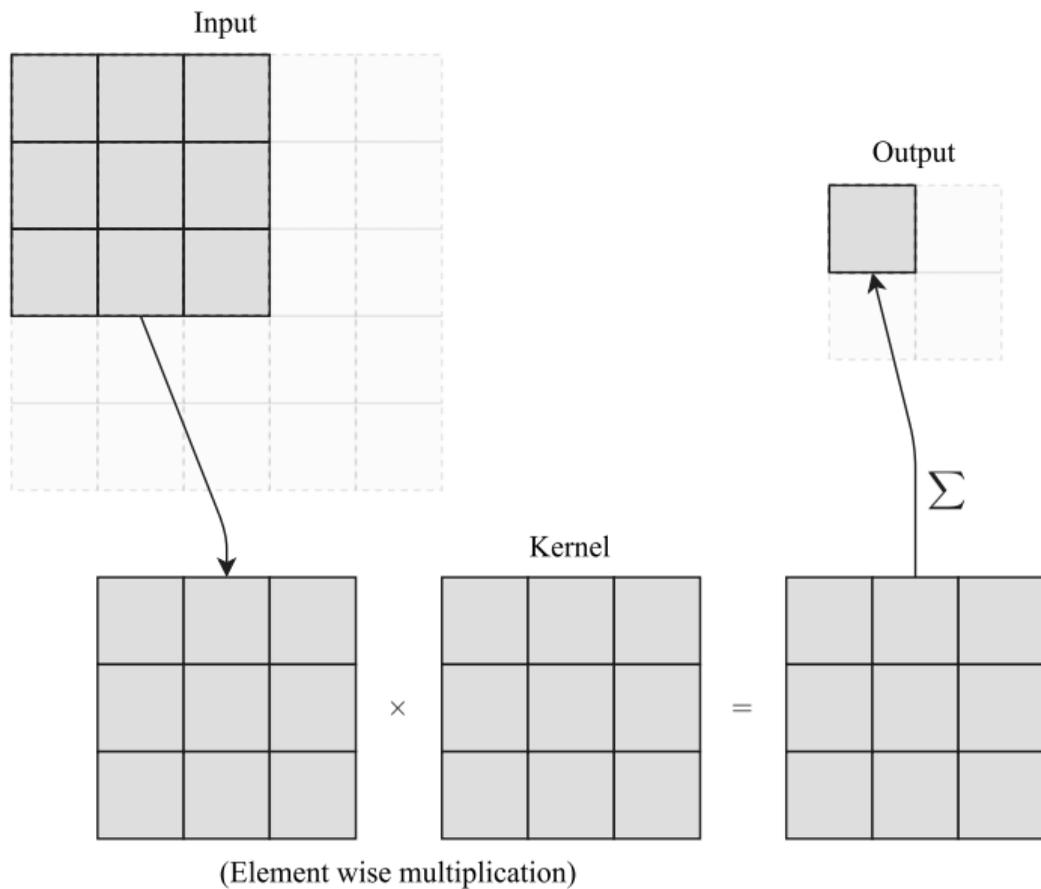
# CNN: convolutional layer



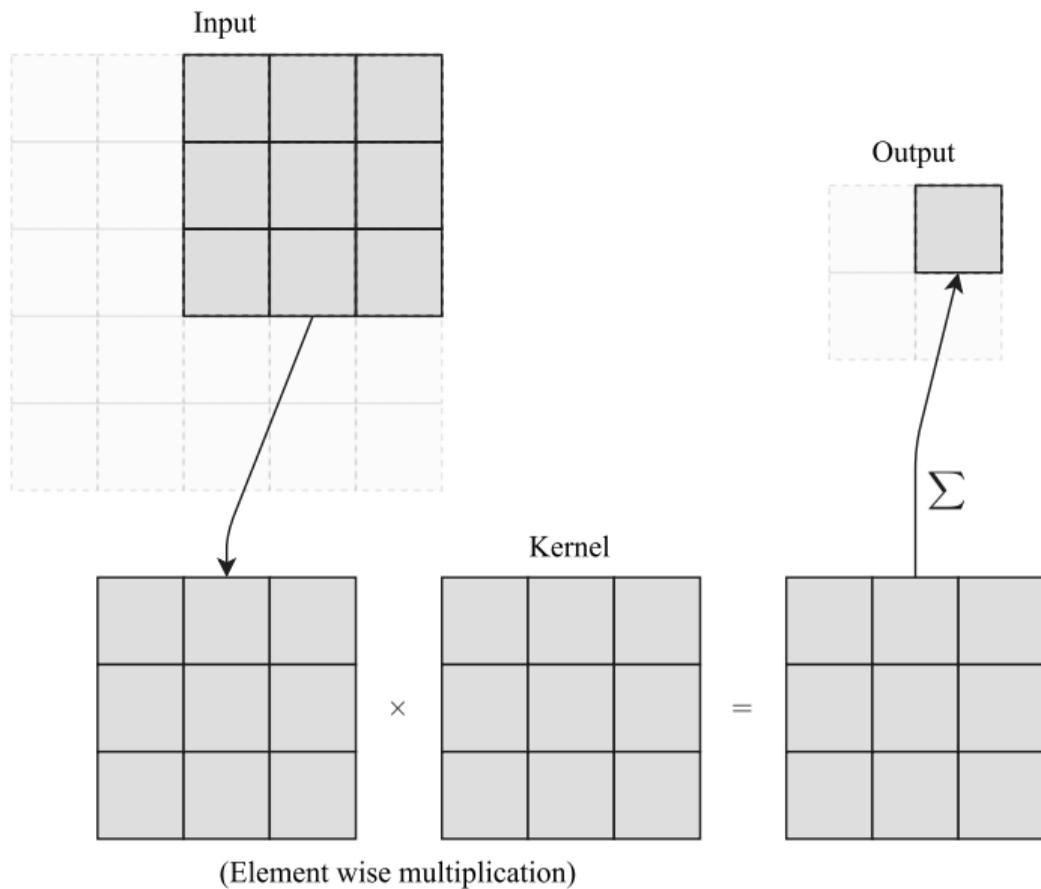
# Convolutional layer - padding



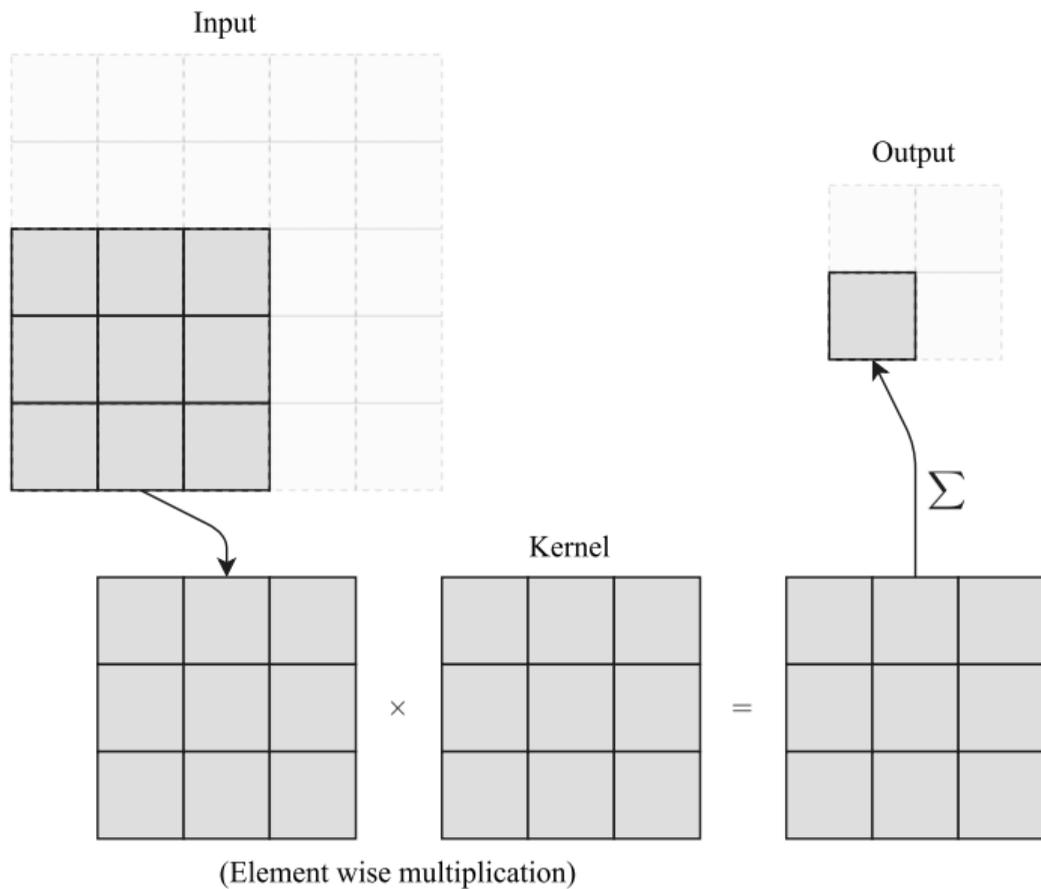
# Convolutional layer - stride



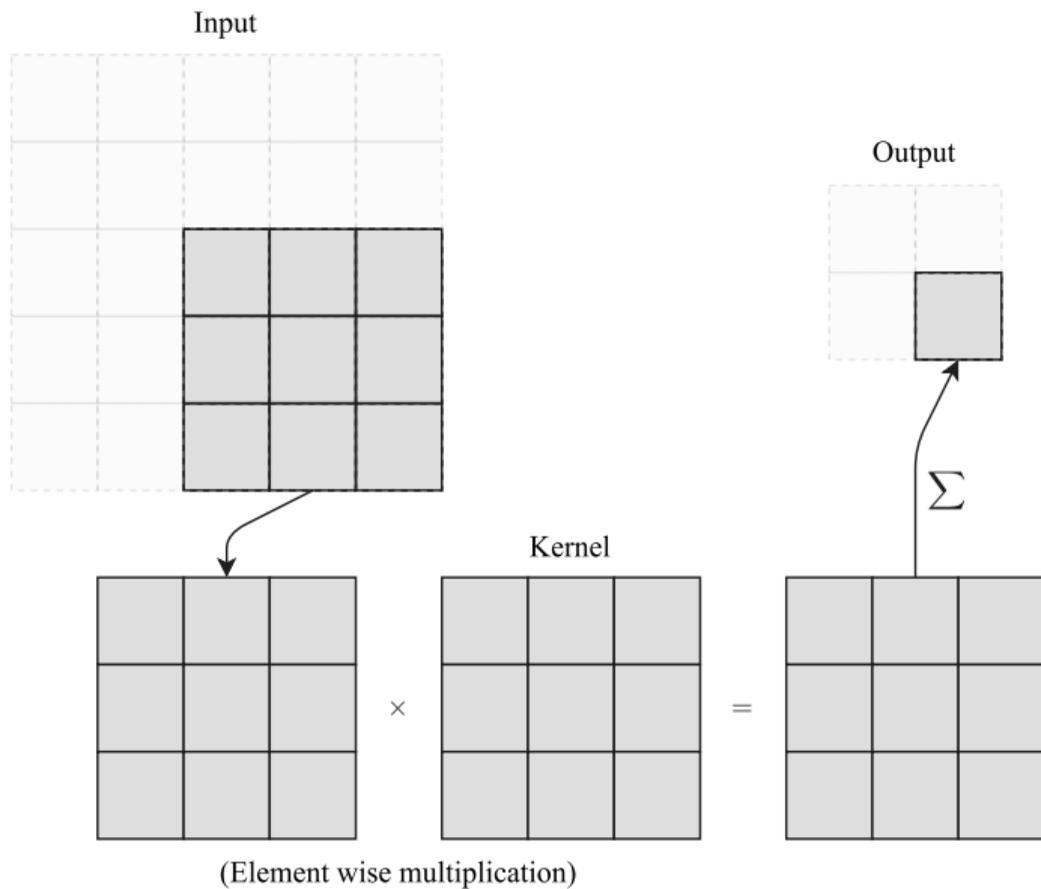
# Convolutional layer - stride



# Convolutional layer - stride



# Convolutional layer - stride



# Convolutional layer - summary

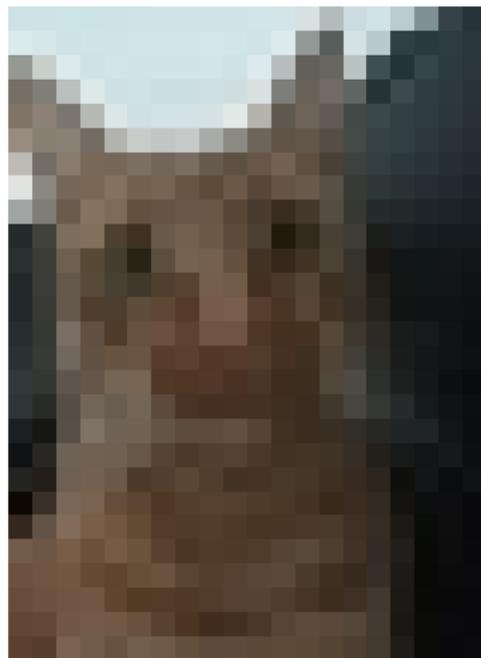
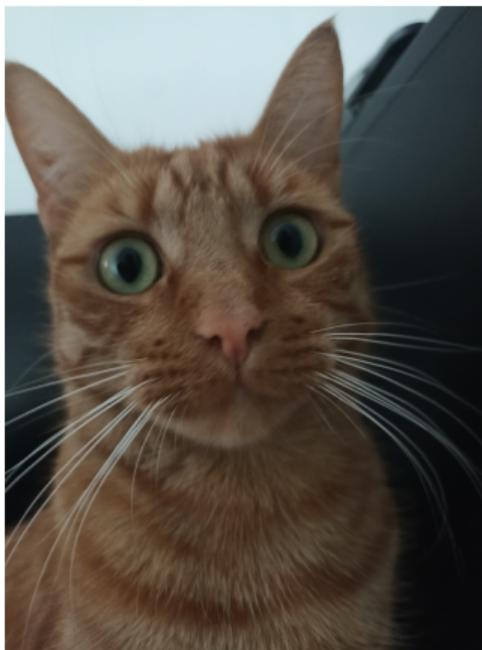
- Input size (of the layer and of every neuron): Channel  $\times$  Width  $\times$  Height
- Output size (of a neuron):

$$\underbrace{W_{out}}_{\text{width or height}} = \left[ \frac{\overbrace{W_{in} + 2 \times padding - kernel\_size}^{\text{Input total width}}}{\underbrace{stride}_{\text{Number of possible kernel positions}}} \right] \underbrace{+1}_{\text{Starting position}}$$

- Output size (of a layer): Number of neurons  $\times W_{out} \times H_{out}$

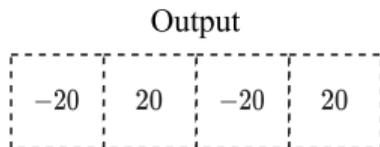
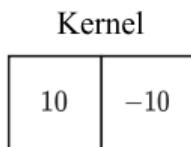
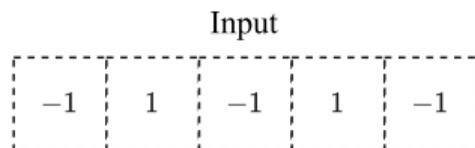
## Pooling layer

A high resolution/dimensionality may not be needed to recognize the content...

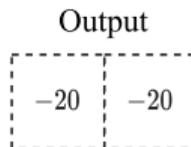
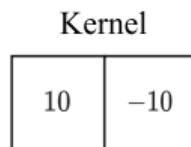
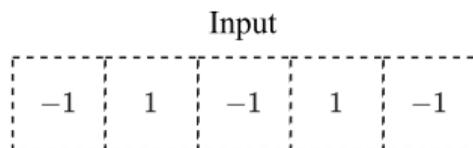


# Pooling layer

A high resolution/dimensionality may not be needed to recognize the content...  
...but increasing the stride can be risky...



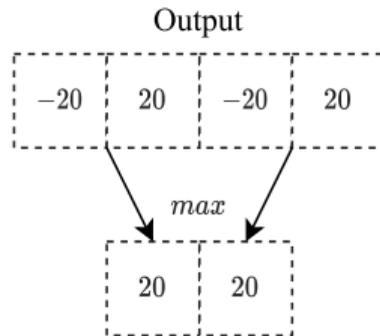
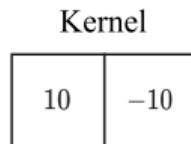
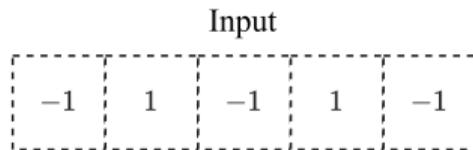
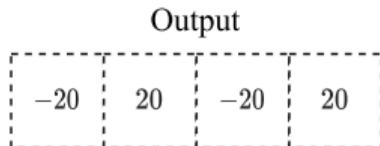
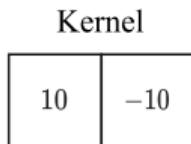
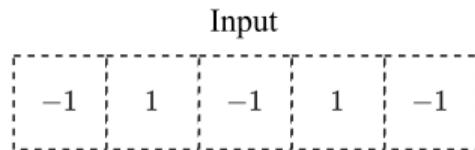
(Stride = 1)



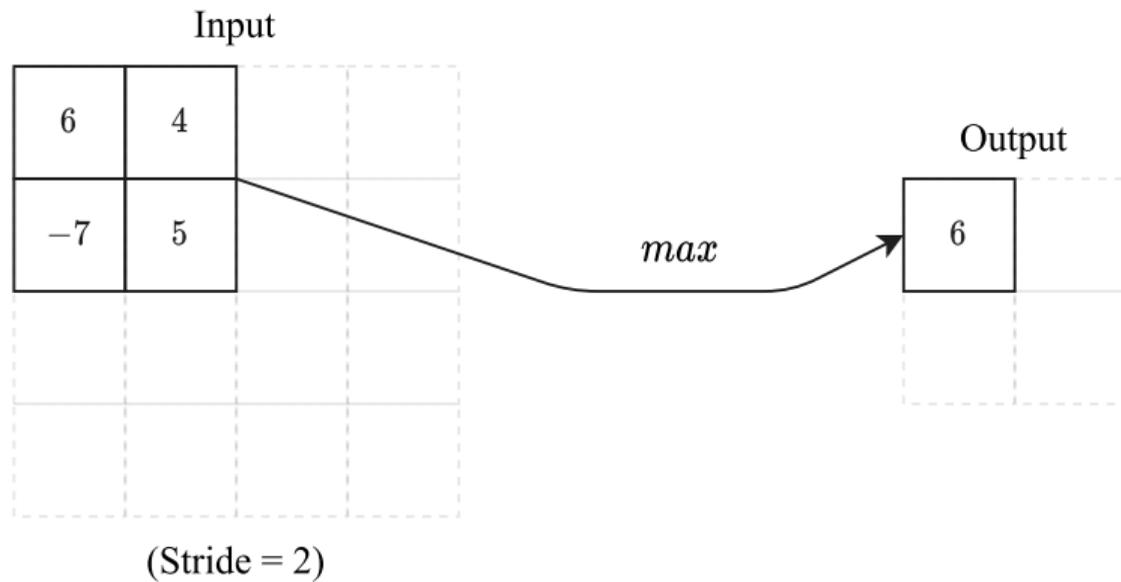
(Stride = 2)

# Pooling layer

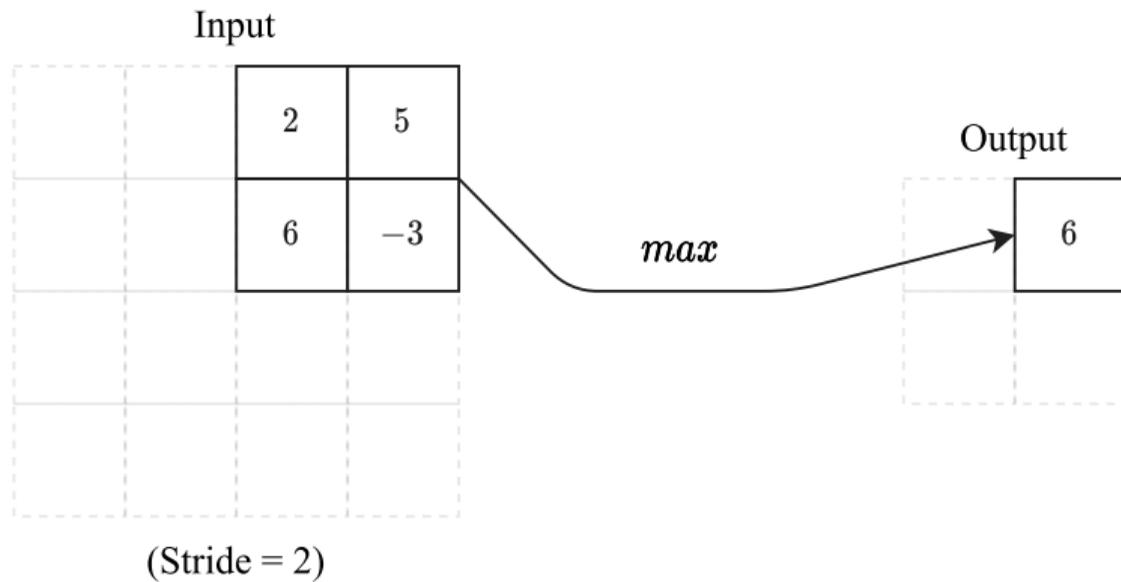
A high resolution/dimensionality may not be needed to recognize the content...  
...but increasing the stride can be risky...  
...so we tend to prefer max or average pooling



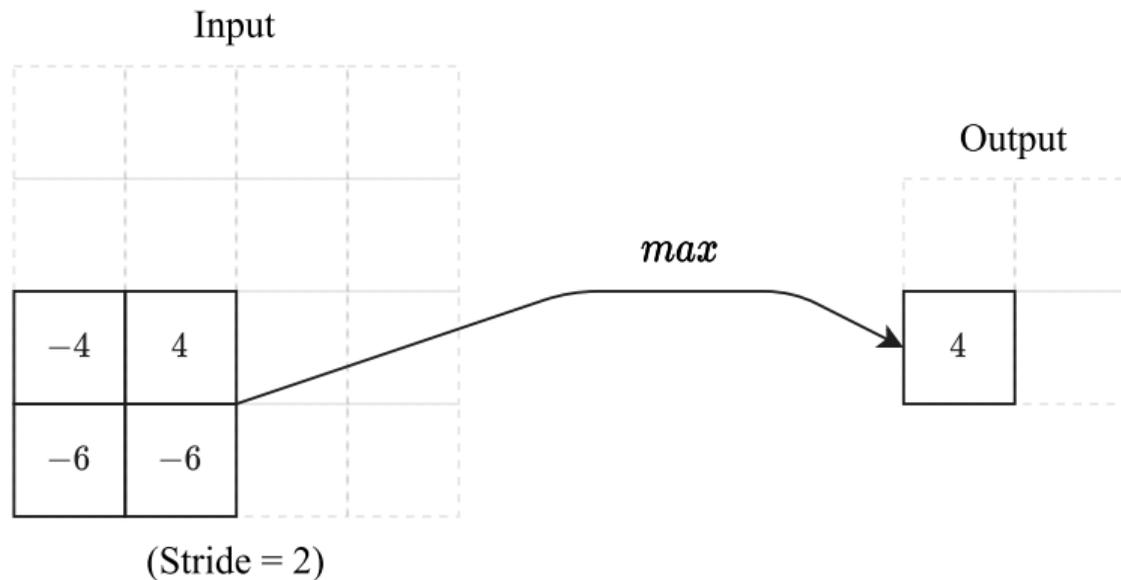
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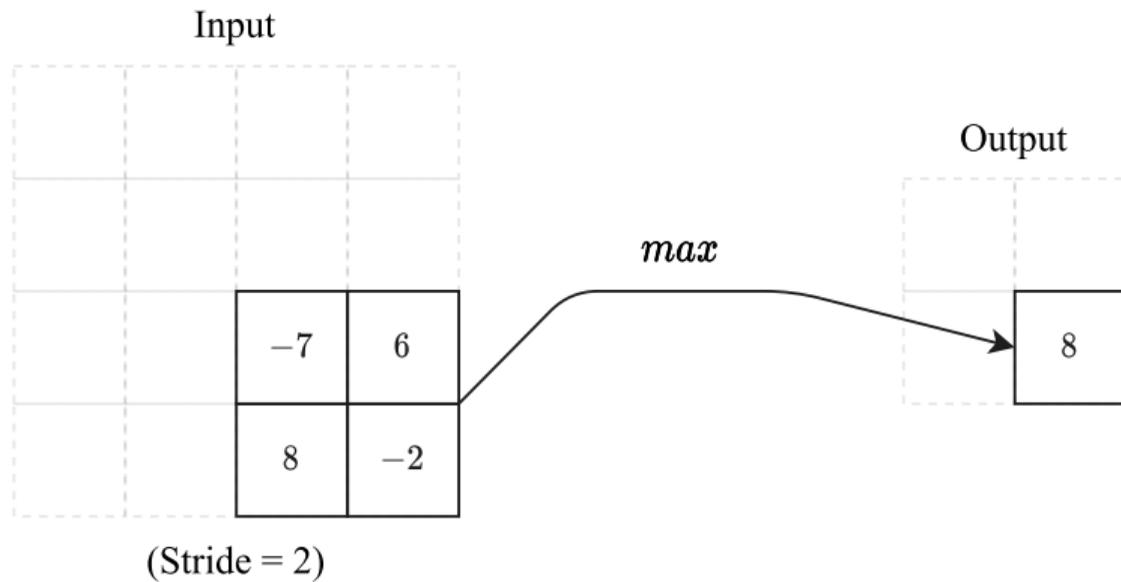
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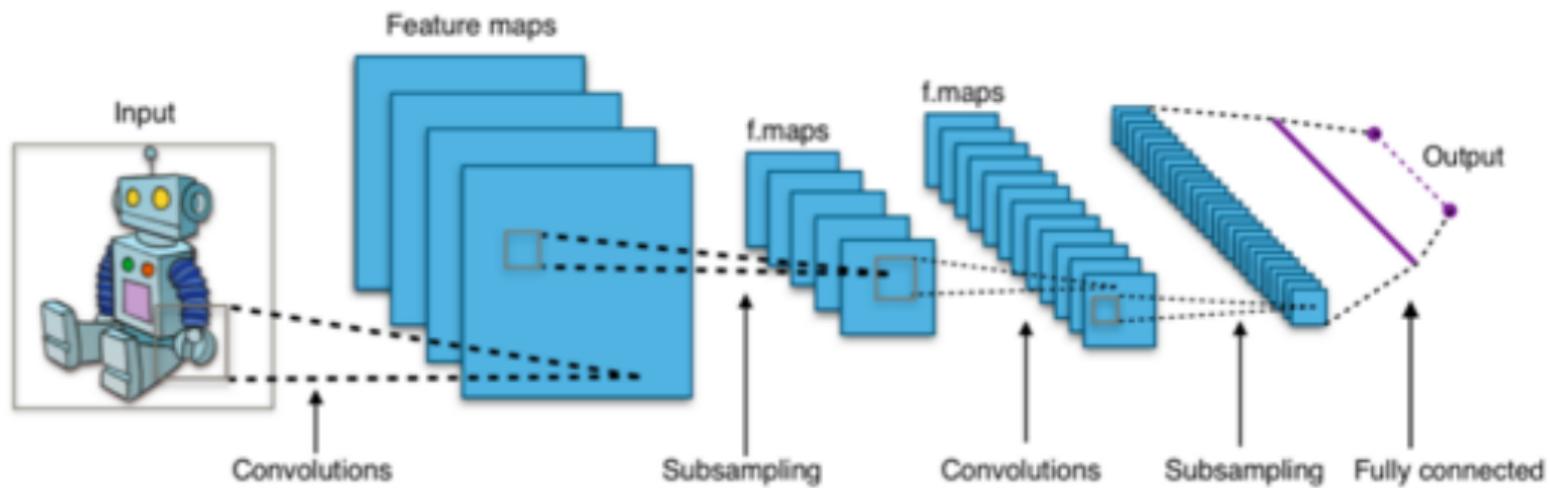


# Pooling layer

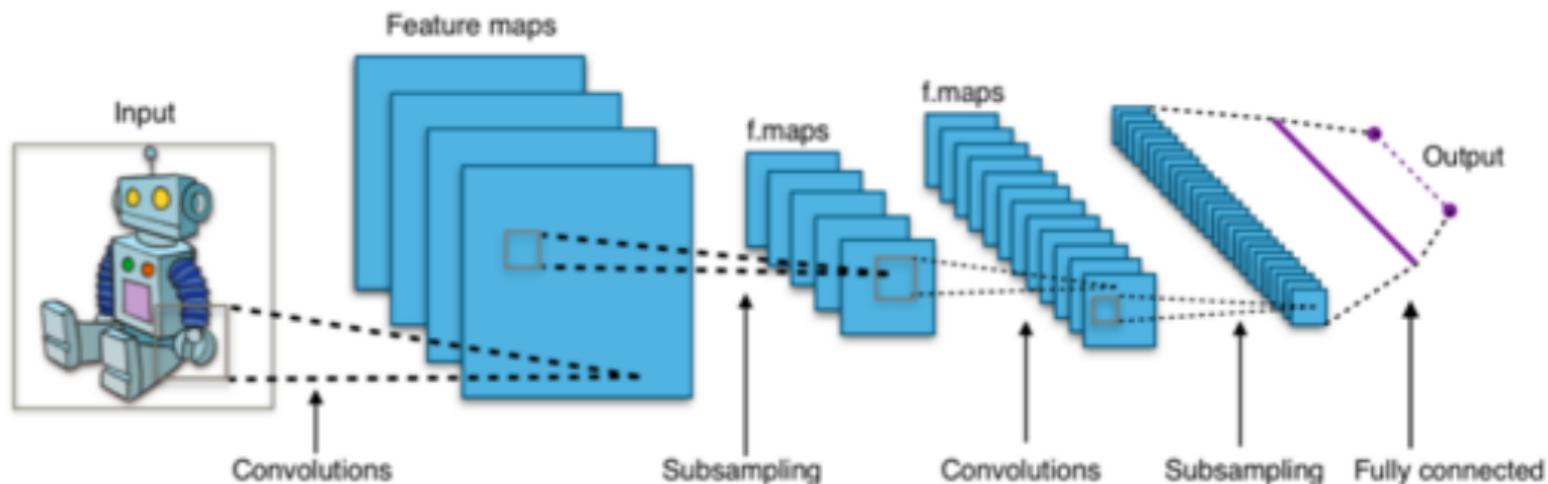
Adding pooling layers helps with:

- removing redundant information
- reducing the amount of computations and memory needed
- making the model more robust to small variations in the input

# CNN - architecture and learning



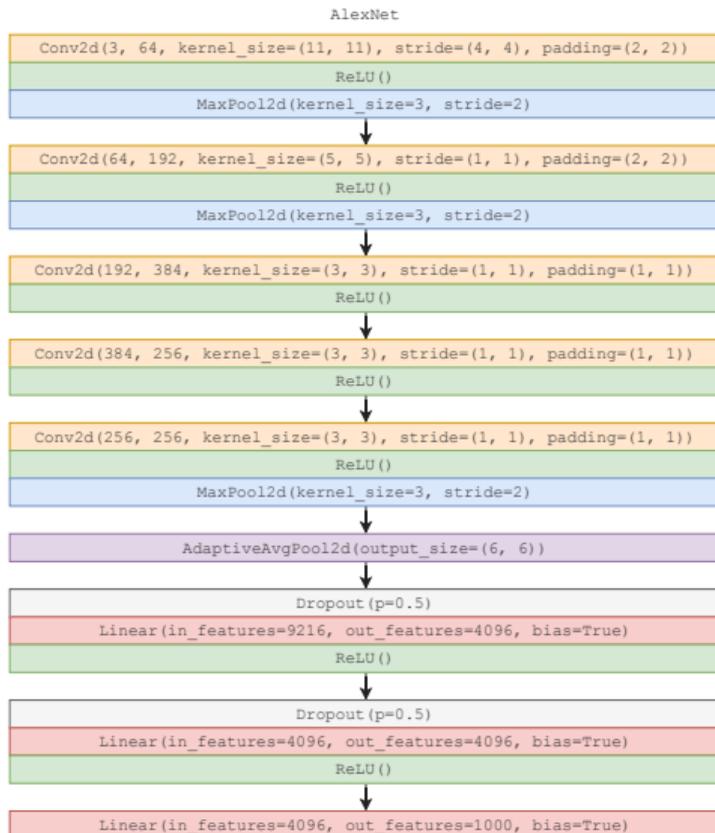
# CNN - architecture and learning



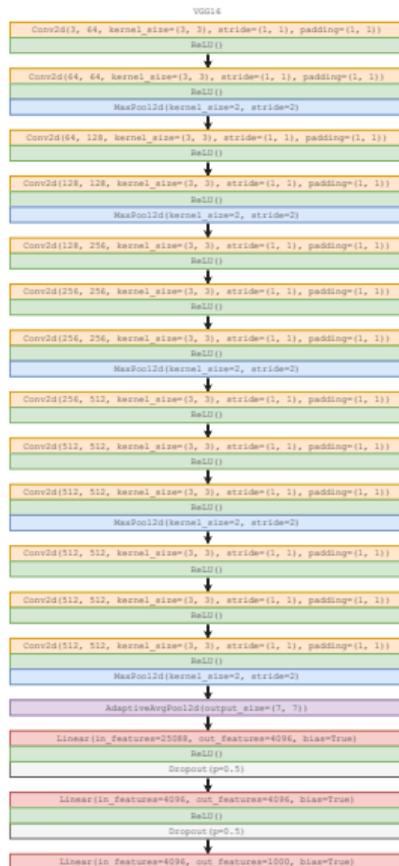
Each neuron does a weighted sum: we can apply SGD on the loss function!

- the weights (kernel) are shared between the neurons of a convolutional layer, so the gradient is aggregated (sum or average)
- for pooling layers, we either backpropagate where the data come from (for max pooling), or do as for any other weighted sum (for average pooling)

# Convolutional Neural Network - AlexNet (2012)



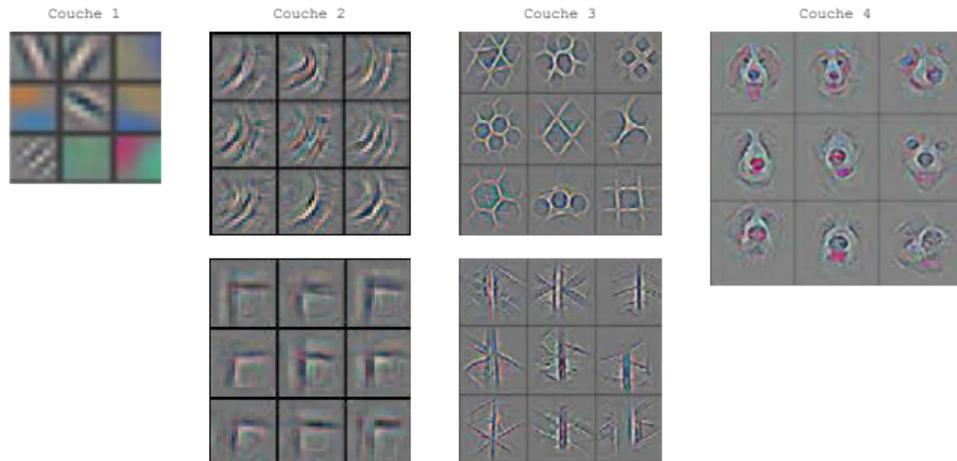
# Convolutional Neural Network - VGG16 (2014)



# CNN summary

## Principles

- Use of convolution for translation invariance and weights sharing
- Pooling to reduce dimensionality
- A MLP at the end of the architecture (no more spatial structure)
- Architecture adaptable to 1D (audio), 3D (video), or graphs...



# Deep learning summary

## Why does it work?

- Universal approximator + a huge amount of data/GPUs... only part of the story
- Hierarchical and automatic learning of features
- Local minima seems quite good [Choromanska et al., 2015]
- Deals better with data/tasks in practice (compared to shallow networks)
- Easy to incorporate inductive biases (e.g., convolution for images)

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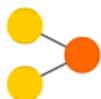
# Deep learning zoo

## A mostly complete chart of Neural Networks

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- Input Cell
- Backfed Input Cell
- △ Noisy Input Cell
- Hidden Cell
- Probabilistic Hidden Cell
- △ Spiking Hidden Cell
- Capsule Cell
- Output Cell
- Match Input Output Cell
- Recurrent Cell
- Memory Cell
- △ Gated Memory Cell
- Kernel
- Convolution or Pool

Perceptron (P)



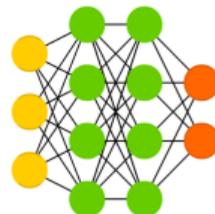
Feed Forward (FF)



Radial Basis Network (RBF)



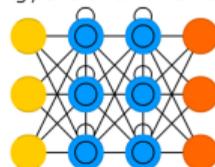
Deep Feed Forward (DFF)



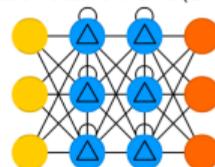
Recurrent Neural Network (RNN)



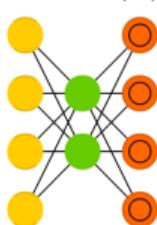
Long / Short Term Memory (LSTM)



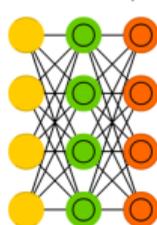
Gated Recurrent Unit (GRU)



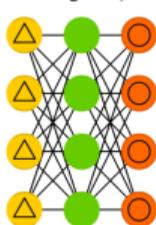
Auto Encoder (AE)



Variational AE (VAE)



Denosing AE (DAE)



Sparse AE (SAE)

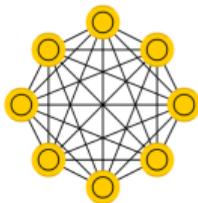


# Deep learning zoo

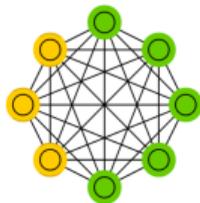
Markov Chain (MC)



Hopfield Network (HN)



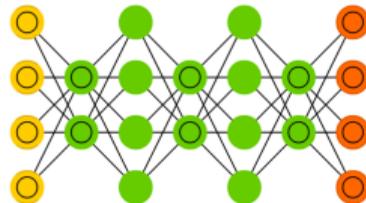
Boltzmann Machine (BM)



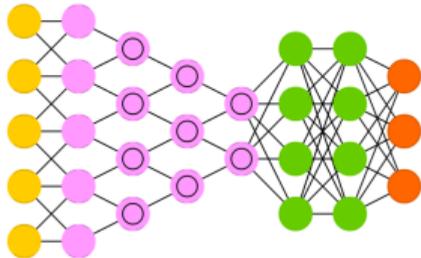
Restricted BM (RBM)



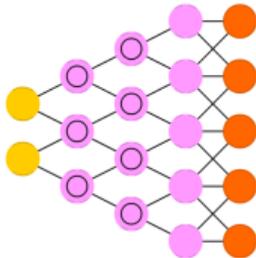
Deep Belief Network (DBN)



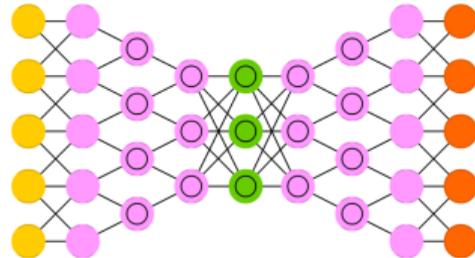
Deep Convolutional Network (DCN)



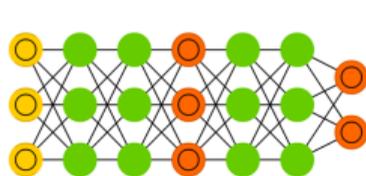
Deconvolutional Network (DN)



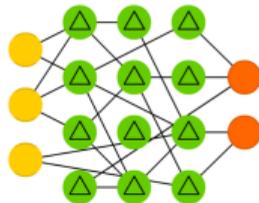
Deep Convolutional Inverse Graphics Network (DCIGN)



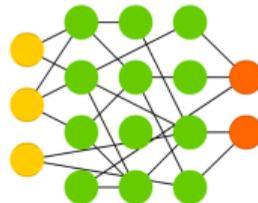
Generative Adversarial Network (GAN)



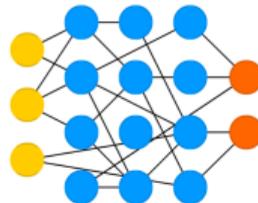
Liquid State Machine (LSM)



Extreme Learning Machine (ELM)

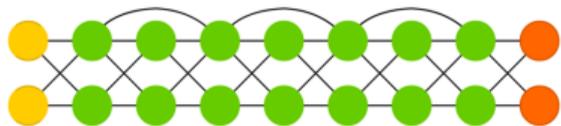


Echo State Network (ESN)

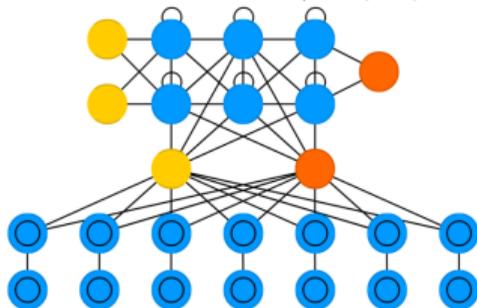


# Deep learning zoo

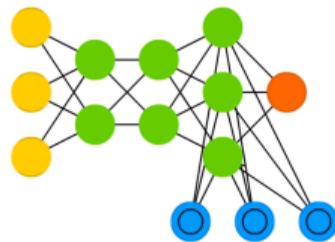
Deep Residual Network (DRN)



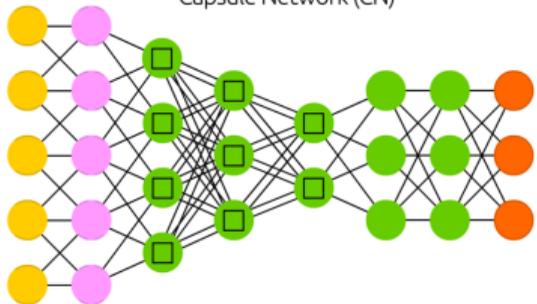
Differentiable Neural Computer (DNC)



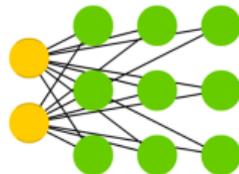
Neural Turing Machine (NTM)



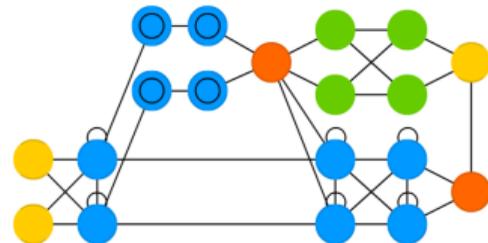
Capsule Network (CN)



Kohonen Network (KN)

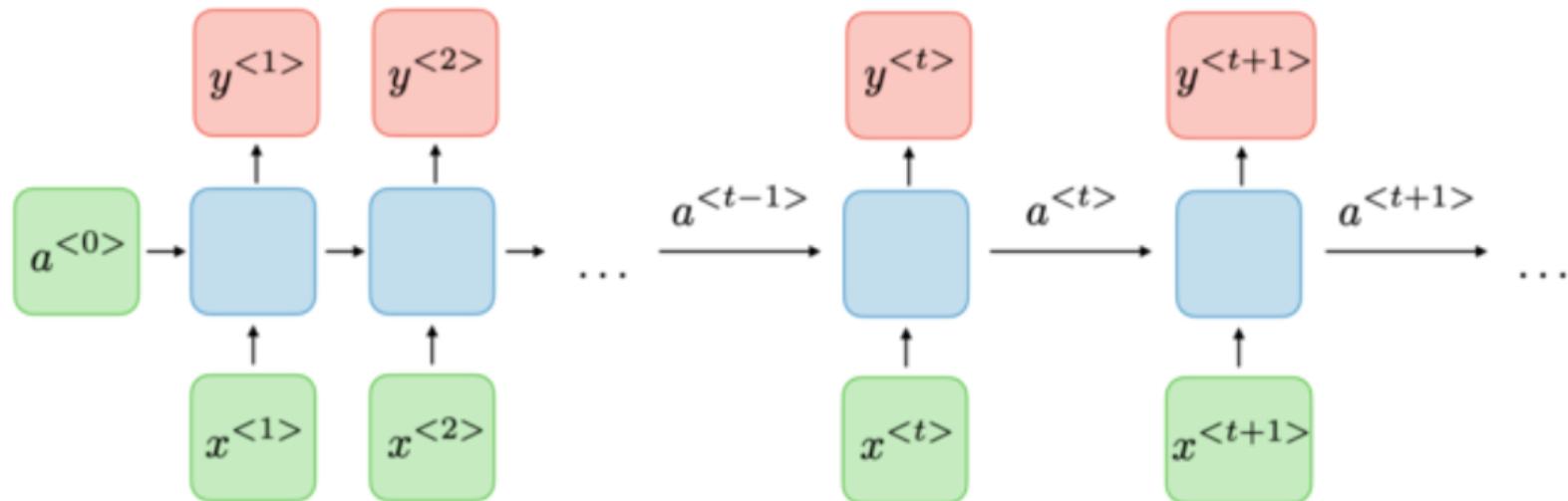


Attention Network (AN)

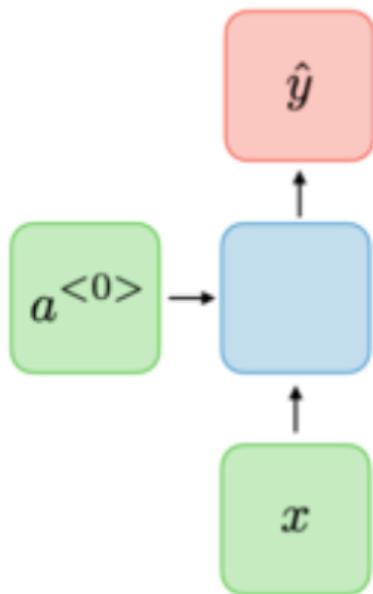


<https://www.asimovinstitute.org/neural-network-zoo/>

# Recurrent neural networks (RNN)

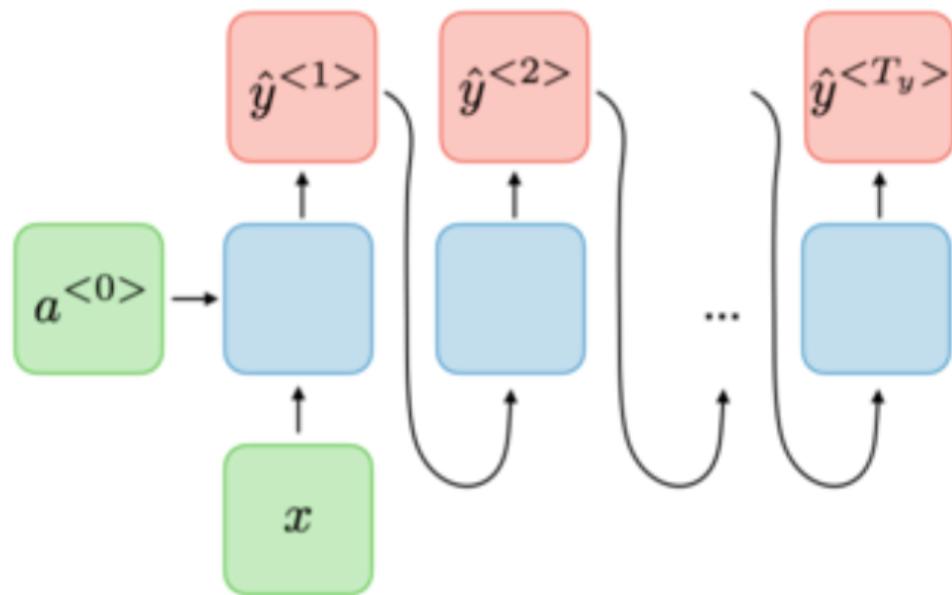


# Recurrent neural networks (RNN) - applications



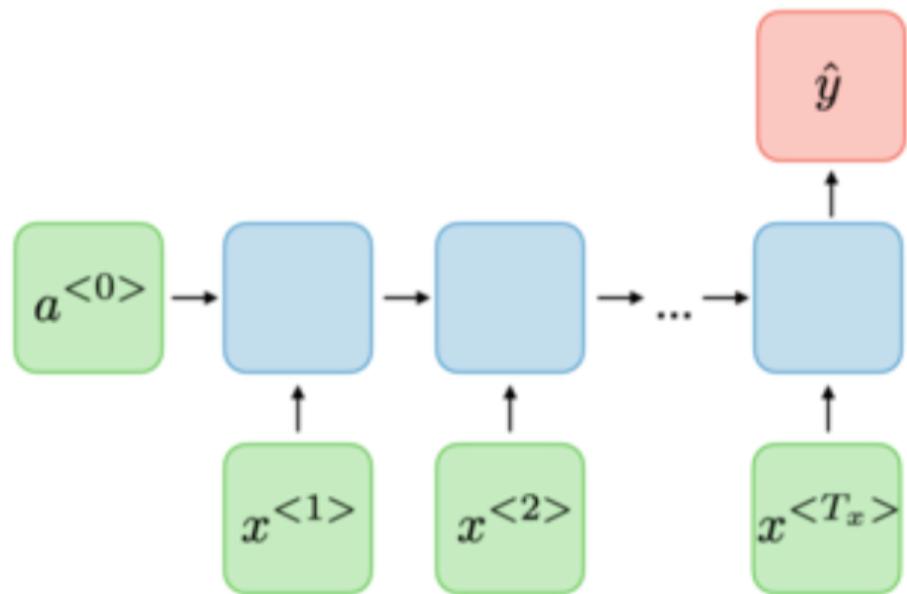
one to one

# Recurrent neural networks (RNN) - applications



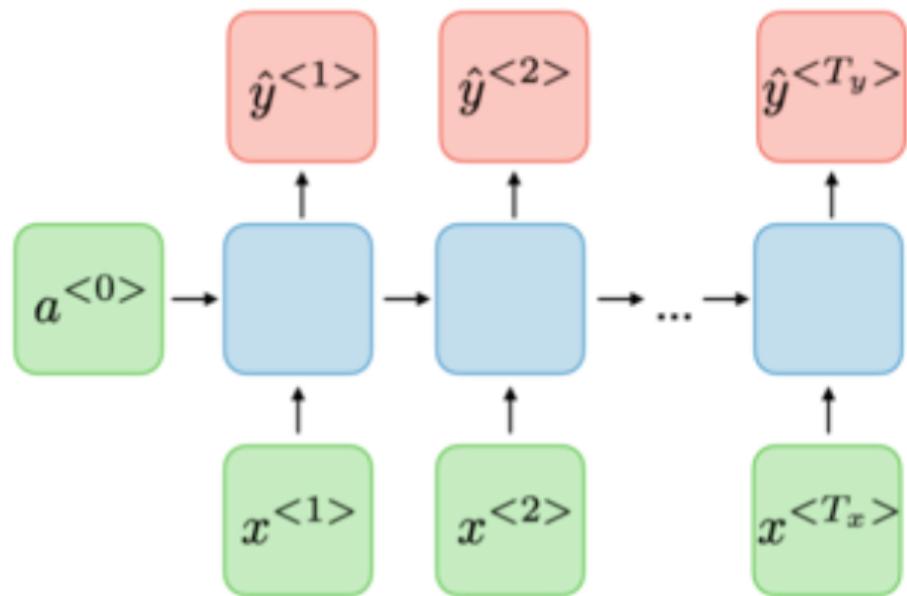
one to many

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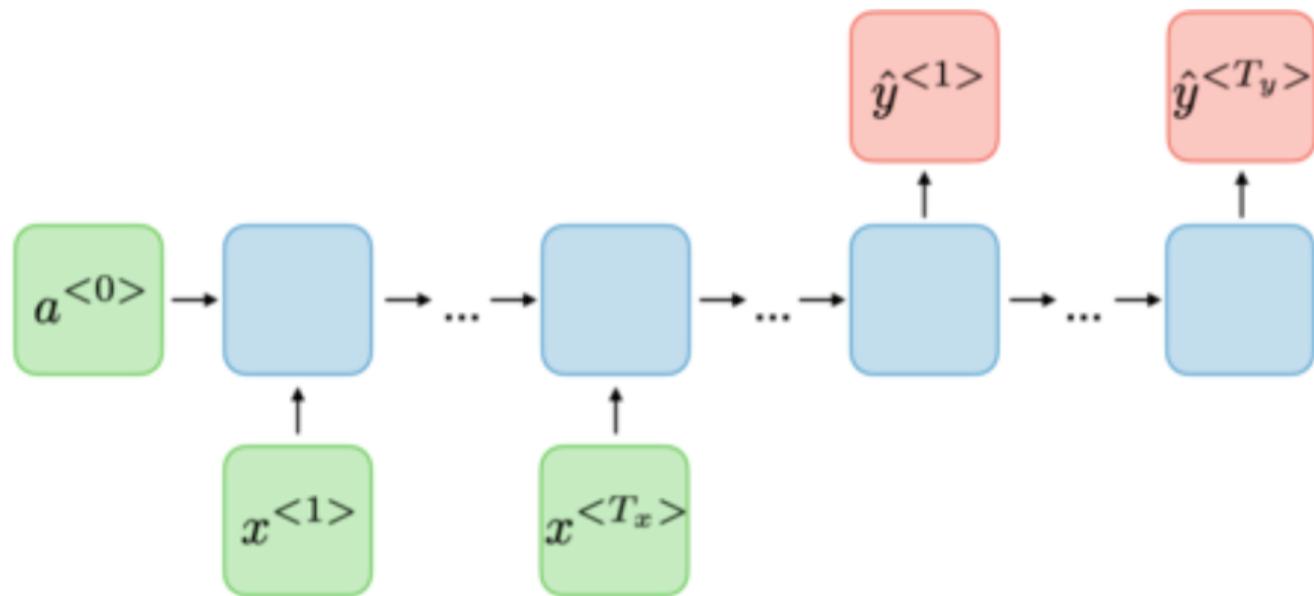
many to one

# Recurrent neural networks (RNN) - applications



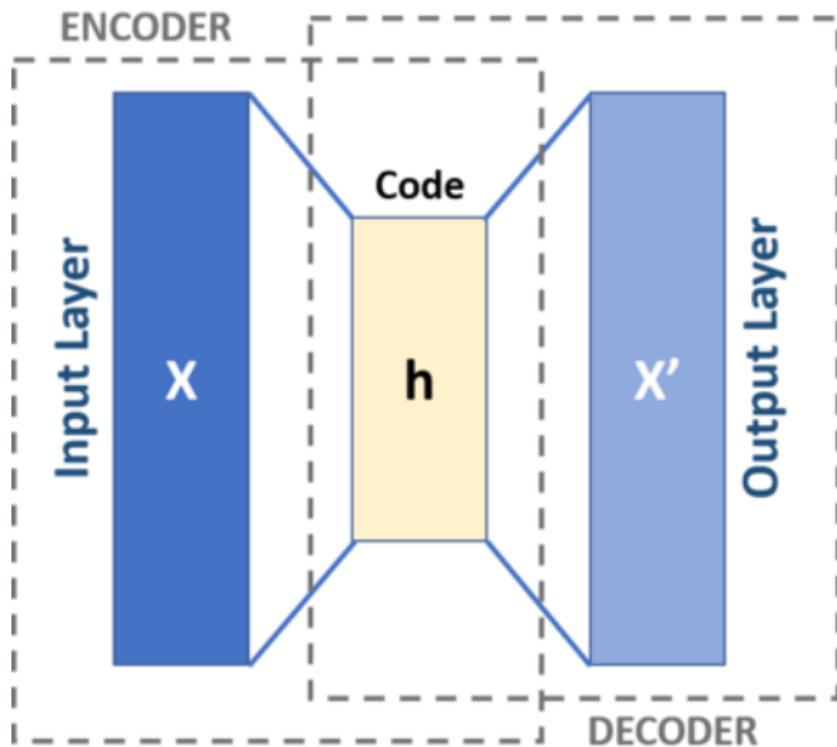
many to many (aligned)

# Recurrent neural networks (RNN) - applications



many to many (split)

# Autoencoders (AE)



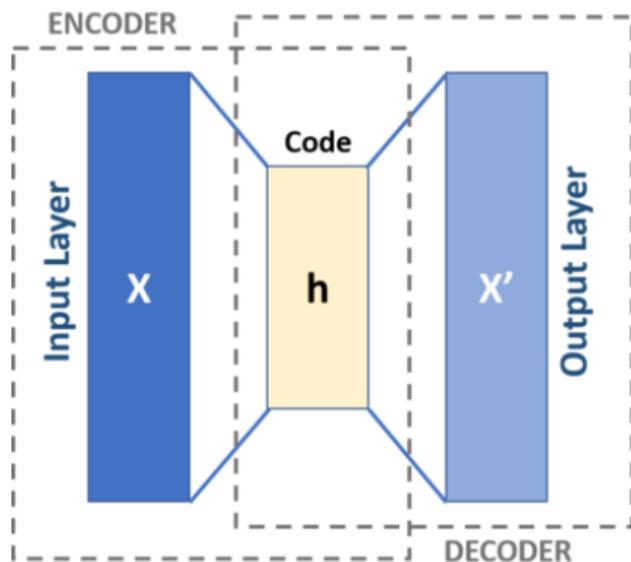
Unsupervised:  $\mathcal{L}(x) = d(x, D(E(x)))$

# Autoencoders (AE) - applications

- Denoising:

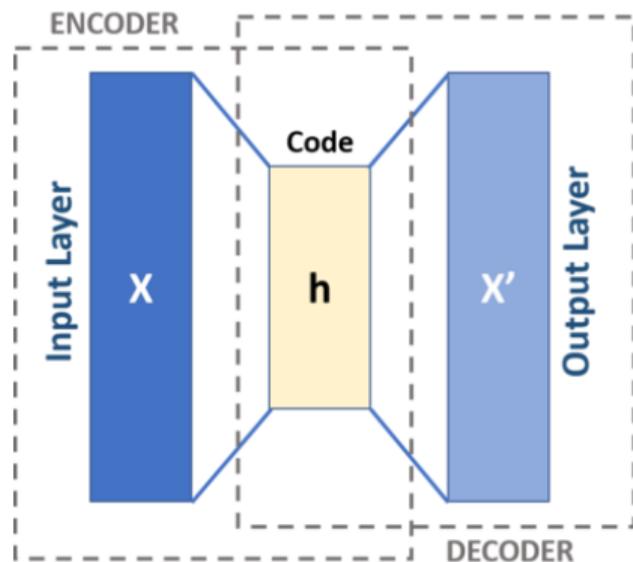
$$\mathcal{L}(x) = d(x, D(E(x + \epsilon)))$$

- Dimensionality reduction using the latent variable/code
- Fraud detection: reconstruction error increases on anomalous data points
- Image compression (convolutional autoencoders)



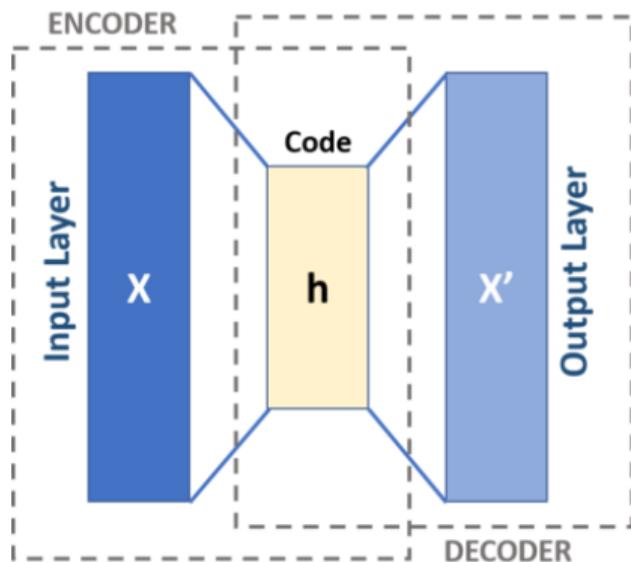
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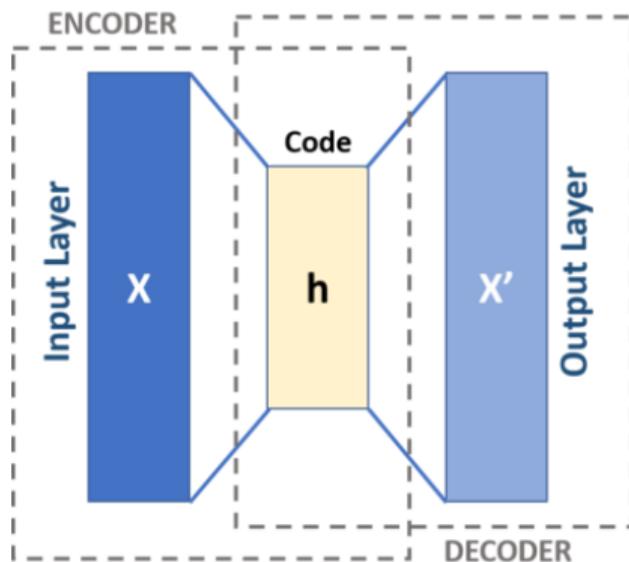
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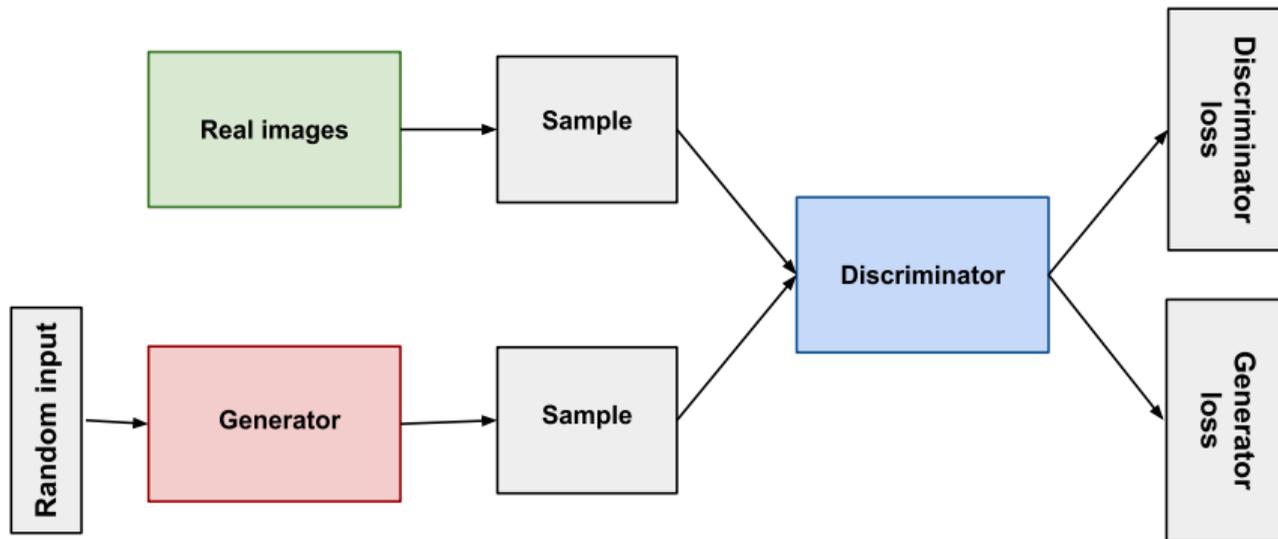


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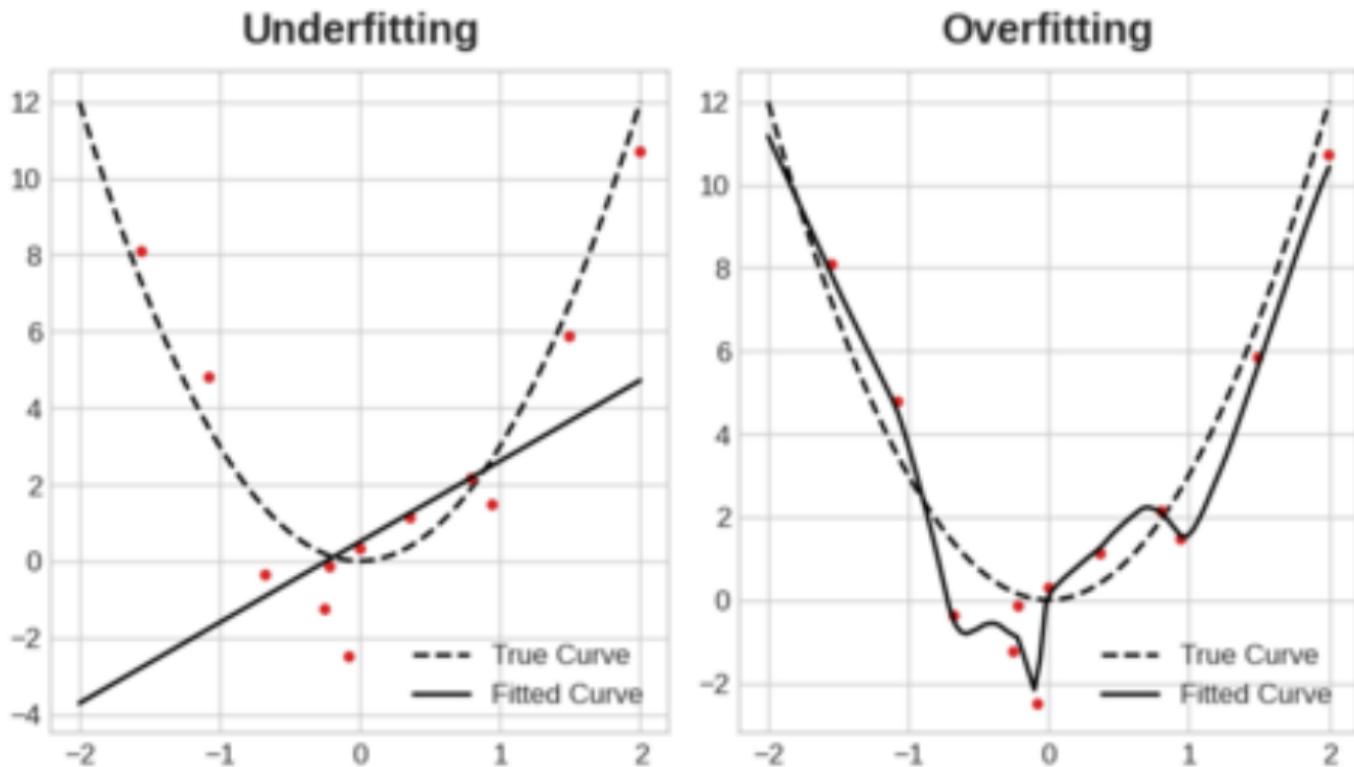


# Generative adversarial networks (GAN)



<https://thispersondoesnotexist.com/>

# Deep learning in practice



→ Regularization!

# Deep learning in practice

- Data (pre) processing
- Choice of the model
- Training
- Getting the better performances

# Deep learning in practice

- Data (pre) processing
  - Data augmentations: e.g., changing the color, zoom, or orientation for images
  - Balanced/representative data
  - Normalizing data:  $\frac{x-\bar{x}}{\sigma(x)}$
  - Use of mini batches
  - Use of train/test/validation datasets
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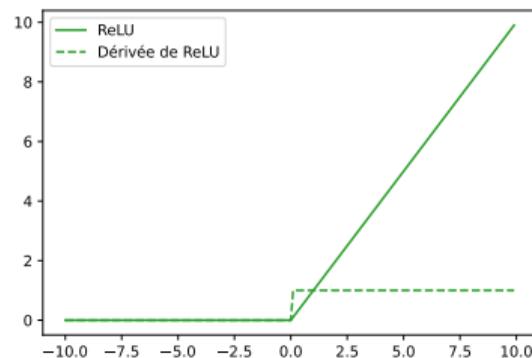
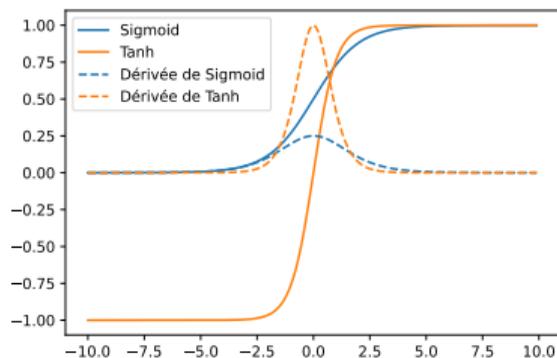
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  - Type of architecture w.r.t. the data / problem
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# Deep learning in practice

- Data (pre) processing
- Choice of the model
- Training

- Loss function:

- regression: (Mean) squared error:  $\frac{1}{2} \sum_{x,i} (y_i - \hat{y}_i)^2$

- classification: softmax + cross entropy:  $\sum_x -\log \frac{e^{\hat{y}_t}}{\sum_i e^{\hat{y}_i}}$

- Regularisation:  $+||\mathbf{w}||$  in the loss function or *drop out* (some weights are randomly set to 0)
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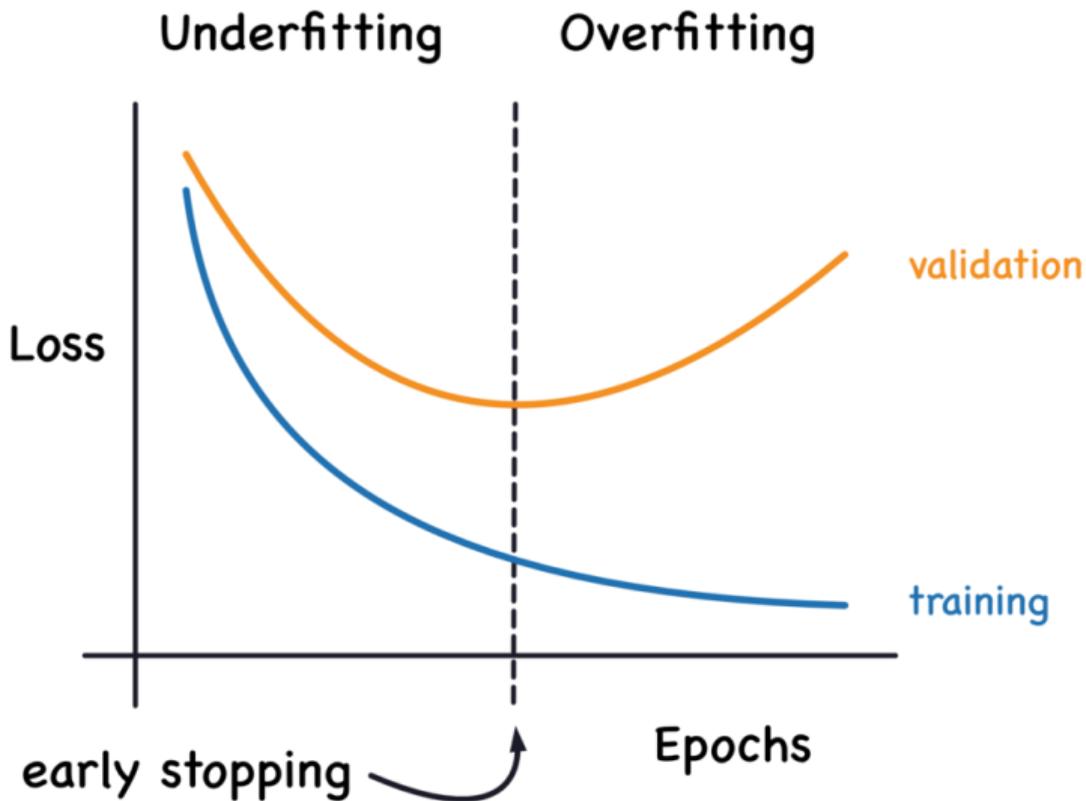
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  - Hyperparameters: start with default values...
    - for the MLP, usually a big 1<sup>st</sup> layer, then decreasing size
    - for the CNN, usually the number of channels increases at each layer (to "compensate" the decreasing size of the feature maps)
  - ...then empirically find what works on the validation set (typically with a *grid search*)
  - *Fine tuning*: use of a ("general") pre trained model that is locally adapted to the data

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## Limitations - Over training?

data aug	dropout	weight decay	top-1 train	top-5 train	top-1 test	top-5 test
ImageNet 1000 classes with the original labels						
yes	yes	yes	92.18	99.21	77.84	93.92
yes	no	no	92.33	99.17	72.95	90.43
no	no	yes	90.60	100.0	67.18 (72.57)	86.44 (91.31)
no	no	no	99.53	100.0	59.80 (63.16)	80.38 (84.49)
Alexnet (Krizhevsky et al., 2012)			-	-	-	83.6
ImageNet 1000 classes with random labels						
no	yes	yes	91.18	97.95	0.09	0.49
no	no	yes	87.81	96.15	0.12	0.50
no	no	no	95.20	99.14	0.11	0.56

Zhang, C., Bengio, S., Hardt, M., Recht, B., & Vinyals, O. (2016). Understanding deep learning requires rethinking generalization. arXiv preprint arXiv:1611.03530.

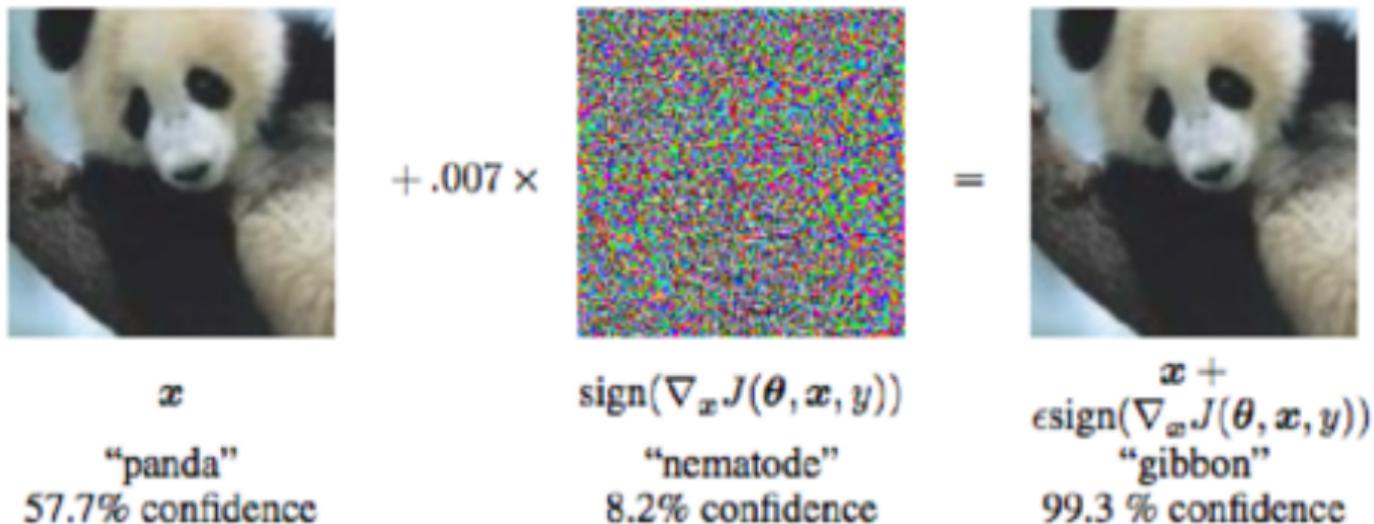
## Limitations - Bias

Classifier	Metric	All	F	M	Darker	Lighter	DF	DM	LF	LM
MSFT	PPV(%)	93.7	89.3	97.4	87.1	99.3	79.2	94.0	98.3	<b>100</b>
	Error Rate(%)	6.3	10.7	2.6	12.9	0.7	<b>20.8</b>	6.0	1.7	0.0
	TPR (%)	93.7	96.5	91.7	87.1	99.3	92.1	83.7	<b>100</b>	98.7
	FPR (%)	6.3	8.3	3.5	12.9	0.7	<b>16.3</b>	7.9	1.3	0.0
Face++	PPV(%)	90.0	78.7	99.3	83.5	95.3	65.5	<b>99.3</b>	94.0	99.2
	Error Rate(%)	10.0	21.3	0.7	16.5	4.7	<b>34.5</b>	0.7	6.0	0.8
	TPR (%)	90.0	98.9	85.1	83.5	95.3	98.8	76.6	<b>98.9</b>	92.9
	FPR (%)	10.0	14.9	1.1	16.5	4.7	<b>23.4</b>	1.2	7.1	1.1
IBM	PPV(%)	87.9	79.7	94.4	77.6	96.8	65.3	88.0	92.9	<b>99.7</b>
	Error Rate(%)	12.1	20.3	5.6	22.4	3.2	<b>34.7</b>	12.0	7.1	0.3
	TPR (%)	87.9	92.1	85.2	77.6	96.8	82.3	74.8	<b>99.6</b>	94.8
	FPR (%)	12.1	14.8	7.9	22.4	3.2	<b>25.2</b>	17.7	5.20	0.4

Buolamwini, Joy, and Timnit Gebru. "Gender shades: Intersectional accuracy disparities in commercial gender classification." Conference on Fairness, Accountability and Transparency. 2018

# Limitations - Adversarial Attacks

Small, often imperceptible, perturbations to the input can cause the model to make a completely wrong prediction with high confidence.



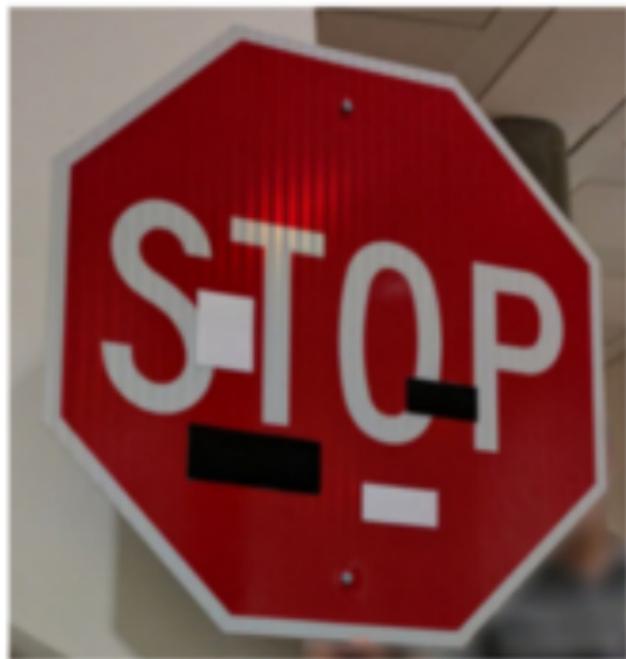
Goodfellow et al. (2014). Explaining and harnessing adversarial examples.

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A physically-realized attack causes the model to classify this stop sign as a "Speed Limit 45" sign.

Eykholt et al. (2018). Robust Physical-World Attacks on Deep Learning.



# Limitations - Environmental impact

- Impact of digital technologies is estimated between 1.5 and 4% of global greenhouse gases emissions (~37 billions of tons eqCO<sub>2</sub> in 2023)
- AI contribution (very) difficult to estimate, but clearly growing.
- For instance (estimations), GPT3 training required 1,287 MWh (~500 tons eqCO<sub>2</sub>)...
- ...but ChatGPT inference needs 564 MWh (~220 tons eqCO<sub>2</sub>) every day (i.e. ~80000 tons eqCO<sub>2</sub> per year).

Freitag, C., Berners-Lee, M., Widdicks, K., Knowles, B., Blair, G., & Friday, A. (2021). The climate impact of ICT: A review of estimates, trends and regulations. arXiv preprint arXiv:2102.02622.

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- Warren S McCulloch and Walter Pitts. A logical calculus of the ideas immanent in nervous activity. *The bulletin of mathematical biophysics*, 5(4): 115–133, 1943.
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